i.e., $F_{t_2} = \dfrac{9550 \times 1000 \times 12 \times 1.5}{2000 \times 48} = 1790.625$ N

b) Tangential tooth load from equation $F_t = \sigma_o$ byp $C_v = \sigma_o$ byp K_v---- 2.93(Old DDHB); 23.93 (New DDHB)

i.e. Tangential tooth load of weaker member $F_{t_2} = \sigma_{02}\, by_2p\, C_v = \sigma_{02}\, by_2p\, K_v$

Face width $b = 3\,\pi m$ to $4\,\pi m$ or $9.5\, m < b < 12.5\, m$2.126 (Old); 23.132 (New DDHB)

Select $b = 10\, m$; circular pitch $p = \pi m$

Mean pitch line velocity of weaker member $v_m = \dfrac{\pi d_2 n_2}{60000} = \dfrac{\pi \times 96 \times 2000}{60000} = 10.053$ m/sec.

Velocity factor $K_v = C_v = \dfrac{4.5}{4.5 + v_m} = \dfrac{4.5}{4.5 + 10.053} = 0.3092$ since $v_m < 12.5$ m/sec

---- 2.129 (Old DDHB); 23.135a (New DDHB)

Lewis form factor for weaker member $y_2 = 0.17 - \dfrac{0.95}{z_2}$ ---- 2.99 (Old DDHB); 23.117 (New DDHB)

$$= 0.17 - \frac{0.95}{\left(\dfrac{d_2}{m}\right)} = 0.17 - \frac{0.95 \times m}{96} = 0.17 - 9.896 \times 10^{-3}\, m$$

Substituting all the values in Lewis equation,

$$1790.625 = (138)(10\,m)(0.17 - 9.896 \times 10^{-3}\, m)(\pi m)(0.3092)$$

$$\text{i.e., } 1.3358 = 0.17\, m^2 - 9.896 \times 10^{-3}\, m^3$$

$$\text{i.e., } m^2 - 0.0582\, m^3 \geq 7.8576$$

Trail : 1

Select module $m = 3$ mm [Select standard module from Table 2.3 (Old DDHB); Table 23.3 (New DDHB)]

$$\text{i.e., } 3^2 - 0.0582 \times 3^3 \geq 7.8576$$

$$\text{i.e., } 7.4286 < 7.8576 \quad \therefore \text{ Not suitable}$$

Trail : 2

Select module $m = 4$mm

$$4^2 - 0.0582 \times 4^3 \geq 7.8576$$

$12.2752 > 7.8576$. Hence suitable

$\therefore$ module $m = 4$mm

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{02} K_v)_{all} = (138.34)(0.3092) = 42.6696$ N/mm^2

Induced stress $\sigma_{ind} = (\sigma_{02} K_v)_{ind} = \dfrac{F_{t_2}}{by_2p}$ ---- 2.93(Old DDHB); 23.93 (New DDHB)

$$= \frac{1790.625}{(10 \times 4)(0.17 - 9.896 \times 10^{-3} \times 4)(\pi \times 4)} = 27.315 \text{ N/mm}^2$$

Since $(\sigma_{02} K_v)_{ind} < (\sigma_{02} K_v)_{all}$, the design is safe. Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth.

$$\text{Effective force } F_{eff} = \frac{F_{t_2}.C_s}{K_v} = \frac{F_{t_2}}{K_v} \quad \text{since } C_s \text{ is already considered.}$$

$$= \frac{1790.625}{0.3092} = 5791.2\,N$$

Beam strength of weaker number $F_{b_2} = \sigma_{02}\, b y_2 p$

$$= (138)(10\times4)(0.17 - 9.896\times10^{-3}\times4)(\pi\times4) = 9046.484\,N$$

$$F_{b_2} > F_{eff}$$

$$\therefore \text{ FOS} = \frac{F_{b_2}}{F_{eff}} = \frac{9046.484}{5791.2} = 1.562$$

The design is satisfactory and hence the module should be equal to 4 mm

$\therefore$ Module $m = 4\text{mm}$

face width $b = 10\times4 = 40\,\text{mm}$

Number of teeth on pinion $z_1 = \frac{d_1}{m} = \frac{64}{4} = 16$

Number of teeth on gear $z_2 = \frac{d_2}{m} = \frac{96}{4} = 24$

Example 4.9

A pair of carefully cut (Class - II) spur gear transmit 20 kW at 230 rpm of the gear. Reduction ratio is 5:1. The pinion is made of cast steel heat treated with allowable stress 197 MN/m². Gear is made of cast iron with allowable stress 56 MN/m². Determine module, face width and number of teeth on pinion and gear. Also suggests suitable surface hardness for the gear pair. Pitch line velocity of pinion is not to exceed 7.5 m/sec.

Data :

$N = 20$; $i = 5$; $n_2 = 230$ rpm; $\sigma_{01} = 197\,MN/m^2 = 197\,N/mm^2$; $\sigma_{02} = 56\,MN/m^2 = 56\,N/mm^2$; $v_m \le 7.5$ m/sec

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$\therefore$ Speed of pinion $n_1 = in_2 = 5\times230 = 1150\,\text{rpm}$

Pitch line velocity of pinion $v_m \le 7.5$ m/sec

$$\text{i.e., } \frac{\pi d_1 n_1}{60000} \le 7.5 \text{ m/sec}$$

$$\text{i.e., } \frac{\pi\times d_1\times1150}{60000} \le 7.5$$

$$\therefore d_1 \le 124.556\text{ mm}$$

Select pitch diameter of pinion $d_1 = 120$ mm

$\therefore$ Pitch diameter of gear $d_2 = id_1 = 5\times120 = 600$ mm

To identify the weaker member temporarily assume

$z_1 = 20 \quad \therefore \quad z_2 = iz_2 = 5 \times 20 = 100$

Assume α = 20° full depth involute

Lewis form factor for 20° full depth involute $y = 0.154 - \frac{0.912}{z}$ ----2.98(Old); 23.116 (New DDHB)

Lewis form factor for pinion $y_1 = 0.154 - \frac{0.912}{z_1} = 0.154 - \frac{0.912}{20} = 0.1084$

Lewis form factor for gear $y_2 = 0.154 - \frac{0.912}{z_2} = 0.154 - \frac{0.912}{100} = 0.14488$

i) Identify the weaker member

Particulars	σ_o N/mm²	y	$\sigma_o y$	Remarks
Pinion	197	0.1084	21.3548	
Gear	56	0.14488	8.11328	Weaker

Since $\sigma_{02}\, y_2 < \sigma_{01}\, y_1$, gear is weaker member. Therefore design should be based on gear.

ii) Design

a) Tangential tooth load $F_t = \frac{9550 \times 1000 \times NC_s}{nr} = \frac{9550 \times 1000 \times PC_s}{nr}$ **where r in mm**

$\therefore$ Tangential tooth load of the weaker member $F_{t_2} = \frac{9550 \times 1000NC_s}{n_2 r_2} = \frac{9550 \times 1000PC_s}{n_2 r_2}$

Assume medium shock and 8-10 hours duty per day

$\therefore$ From Table 2.33 (Old DDHB); Table 23.13 (New DDHB) (Page 23.76), service factor $C_s = 1.5$

Pitch circle radius of gear $r_2 = \frac{d_2}{2} = \frac{600}{2} = 300$ mm

i.e., $F_{t_2} = \frac{9550 \times 1000 \times 20 \times 1.5}{230 \times 300} = 4152.2$ N

b) Tangential tooth load from Lewis equation $F_t = \sigma_o\, byp\, C_v = \sigma_o\, byp\, K_v$ **---- 2.93(Old DDHB); 23.93 (New DDHB)**

Tangential tooth load of the weaker member $F_{t_2} = \sigma_{02}\, by_2 p\, C_v = \sigma_{02}\, by_2 p\, K_v$

Face width $b = 3\pi m$ to $4\pi m$ or $9.5\, m < b < 12.5\, m$ ----2.126(Old); 23.132 (New DDHB)

$\therefore$ Select $b = 10$ m ; circular pitch $p = \pi m$

$y_2 = 0.154 - \frac{0.912}{z_2} = 0.154 - \frac{0.912}{\frac{d_2}{m}} = 0.154 - \frac{0.912m}{600} = 0.154 - 1.52 \times 10^{-3}\, m$

Mean pitch line velocity of weaker member $v_m = \frac{\pi d_2 n_2}{60000} = \frac{\pi \times 600 \times 230}{60000} = 7.2257$ m/sec

$\therefore$ Velocity factor $K_v = \dfrac{3}{3+v_m} = \dfrac{3}{3+7.2257} = 0.2934$ since $v_m < 7.5$ m/sec

---- 2.128(Old DDHB); 23.134a (New DDHB)

Substituting all the values in Lewis equation

$$4152.2 = (56)(10\,m)(0.154 - 1.52\times 10^{-3}m)(\pi m)(0.2934)$$

$$\text{i.e., } 8.045 = 0.154\,m^2 - 1.52\times 10^{-3}m^3$$

i.e., $m^2 - 9.87\times 10^{-3}m^3 - 52.2384 = 0$

i.e., $m^2 - 9.87\times 10^{-3}m^3 \geq 52.2384$

Trail : 1

Select module $m = 6$ mm [Select standard module from Table 2.3 (Old DDHB); Table23.3 (New DDHB)]

i.e., $6^2 - 9.87\times 10^{-3}\times 6^3 \geq 52.2384$; i.e., $33.868 < 52.2384$

$\therefore$ Not suitable

Trail : 2

Select module $m = 8$mm

i.e., $8^2 - 9.87\times 10^{-3}\times 8^3 \geq 52.2384$

i.e., $58.9466 > 52.2384$

Hence suitable

$\therefore$ Module $m = 8$mm

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{02} K_v) = 56\times 0.2934 = 16.4304$ N/mm^2

Induced stress $\sigma_{ind} = (\sigma_{02} K_v) = \dfrac{F_{t_2}}{by_2p} = \dfrac{4152.2}{(10\times 8)(0.154 - 1.52\times 10^{-3}\times 8)(\pi\times 8)} = 14.56$ N/mm^2

---- 2.93(Old DDHB); 23.93 (New DDHB)

Since $(\sigma_{02} K_v)_{ind} < (\sigma_{02} K_v)_{all}$, the design is safe. Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth.

Effective force $F_{eff} = \dfrac{F_{t_2} C_s}{K_v} = \dfrac{F_{t_2}}{K_v}$ since C_s in already considered $= \dfrac{4152.2}{0.2934} = 14152$ N

Beam strength of weaker member $F_{b_2} = \sigma_{02}\, by_2p$

$= (56)(10\times 8)(0.154 - 1.52\times 10^{-3}\times 8)(\pi\times 8) = 15970.43$ N

$$F_{b_2} > F_{eff}$$

$$\therefore \text{ FOS} = \frac{F_{b_2}}{F_{eff}} = \frac{15970.43}{14152} = 1.13$$

The design is satisfactory and hence the module should be equal to 8 mm

$\therefore$ Module $m = 8$ mm

Face width $b = 10\,m = 10\times 8 = 80$ mm

Number of teeth on pinion $z_1 = \dfrac{d_1}{m} = \dfrac{120}{8} = 15$

Number of teeth on gear $z_2 = \frac{d_2}{m} = \frac{600}{8} = 75$ ---- 2.160(Old DDHB); 23.160 (New DDHB)

iii) Checking

a) Dynamic load $F_d = F_t + \frac{21v_m \times (F_t + bC)}{21v_m + \sqrt{F_t + bC}}$

Tangential tooth load $F_t = F_{t2} = 4152.2$

Mean pitch line velocity $v_m = 7.2257$ m/sec

Face width $b = 80$ mm

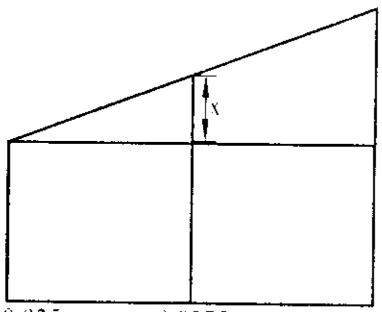
From Fig. 2.29 (Old DDHB) Fig. 23.34a (New DDHB) for carefully cut gear when m = 8 mm, Error f = 0.0375 mm

From Table 2.35 (Old DDHB); Table 23.32 (New DDHB) for $\alpha = 20°$ full depth, steel - CI combination

For f = 0.025 mm; C = 192.14 KN/m = 192.14 N/mm

For f = 0.05 mm; C = 398.28 KN/m = 398.28 N/mm

By interpolation



$$\frac{x}{398.28 - 192.14} = \frac{0.0375 - 0.025}{0.05 - 0.025}$$

$\therefore$ x = 103.07 N/mm

$\therefore$ For error f = 0.0375 mm;

C = 192.14 + 103.07

= 295.21 N/mm

i.e., $F_d = 4152.2 + \frac{21 \times 7.2257[4152.2 + 80 \times 295.21]}{21 \times 7.2257 + \sqrt{4152.2 + 80 \times 295.21}} = 17386.9$ N

b) Wear load $F_w = d_1 bQK$ **---- 2.160(Old DDHB); 23.160 (New DDHB)**

Ratio factor $Q = \frac{2z_2}{z_1 + z_2} = \frac{2 \times 75}{15 \times 75} = 1.667$

For safer design

$F_w \geq F_d$

i.e., $d_1\, bQK \geq F_d$

i.e., (120) (80) (1.667) K $\geq$ 17386.9

$\therefore$ $K \geq 1.0865$ N/mm²

From Table 2.40 (Old DDHB); Table 23.37B (New DDHB) for $\alpha = 20$ FD and $K \geq 1.0865$N/mm²

Surface hardness for pinion = 300 BHN

Surface hardness for gear = 250 BHN

Example 4.10 :

Design a pair of spur gears to transmit a power of 18 kW from a shaft running at 1000 rpm to a parallel shaft to be run at 250 rpm maintaining a distance of 160 mm between the shaft centers. Suggest suitable surface hardness for the gear pair. (VTU Jan/Feb 2005, Jan / Feb 2006)

Data:

$$N = 18\text{ kW};\ n_1 = 1000\text{ rpm};\ n_2 = 250\text{ rpm};\ a = 160\text{ mm}$$

Solution :

Assume

(i) CLASS - III Precision gears

(ii) Pressure angle $\alpha = 20°$ full depth involute system.

(iii) Medium shock and 8 to 10 hours duty per day.

$\therefore$ From Table 23.13 (New DDHB), service factor $C_s = 1.5$

(iv) Pinion material SAE 3245 ($C_r - N_i$ steel) as the centre distance is small

Table 23.18 (New DDHB), $\sigma_{0_1} = 448 - 517$ MPa

$$\text{Take } \sigma_{0_1} = 500\text{ MPa}$$

$$i = \frac{n_1}{n_2} = \frac{1000}{250} = 4 = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$a = \frac{d_1 + d_2}{2} = \frac{d_1 + id_1}{2} = \frac{d_1(1+i)}{2}$$

$$\text{i.e., } 160 = \frac{(1+4)d_1}{2};\ \therefore d_1 = 64\text{mm and } d_2 = 256\text{ mm}$$

Lewis form factor for 20° FD involute system $y = 0.154 - \frac{0.912}{z}$ ----2.98 (Old) ; 23.116 (New DDHB)

To identity the weaker member temporarily assume $z_1 = 20$

$$\therefore z_2 = iz_1 = 4 \times 20 = 80$$

$$\therefore y_1 = 0.154 - \frac{0.912}{z_1} = 0.154 - \frac{0.912}{20} = 0.1084$$

$$y_2 = 0.154 - \frac{0.912}{z_2} = 0.154 - \frac{0.912}{80} = 0.1426$$

To select the gear material equate $\sigma_{0_1} y_1$ to $\sigma_{0_2} y_2$

$$\text{i.e., } 500 \times 0.1084 = \sigma_{0_2} \times 0.1426$$

$$\therefore \sigma_{0_2} = 380.084\text{ N / mm}^2$$

From Table 23.18 (New DDHB), Select the gear material such that its value of σ_{0_2} must be nearer to 380.084 N/mm^2.

Hence select SAE 4640 Hardened by OQT as gear material

$$\therefore \sigma_{0_2} = 379\text{ N / mm}^2$$

(i) Identify the weaker member

Particulars	σ_0, N/mm^2	y	$\sigma_0 y$	Remarks
Pinion	500	0.1084	54.21	
Gear	379	0.1426	54.045	Weaker

As $\sigma_{0_2} y_2 < \sigma_{0_1} y_1$, gear is the weaker member. Therefore design should be based on gear.

(ii) Design

(a) Tangential tooth load $F_t = \dfrac{4550\times1000\times NC_s}{n_r} = \dfrac{9550\times1000\times PC_s}{n_r}$ where r in mm

$\therefore$ Tangential tooth load of weaker member $F_{t_2} = \dfrac{9550\times1000\times NC_s}{n_2 r_2} = \dfrac{9550\times1000\times PC_s}{n_2 r_2}$

Assume medium shock and 8-10 hrs duty per day

$\therefore$ From Table 2.33 (Old DDHB) : Table 23.13 (New DDHB) (Page 23.76) Service factor $C_s = 1.5$

Pitch circle radius of gear $r_2 = \dfrac{d_2}{2} = \dfrac{256}{2} = 128$ mm

$$\therefore F_{t_2} = \frac{9550\times1000\times18\times1.5}{250\times128} = 8057.81 \text{ N}$$

(b) Tangential tooth load from Lewis equation

$F_t = \sigma_0 b y p K_v$ .... 2.93 (Old DDHB) ; 23.93 (New DDHB)

Tangential tooth load of the weaker member $F_{t_2} = \sigma_{0_2} b y_2 p C_v = \sigma_{0_2} b y_2 p K_v$

Face width $b = 3\pi m$ to $4\pi m$ or $9.5\text{ m} < b < 12.5\text{ m}$ .... 2.126 (Old DDHB) ; 23.132 (New DDHB)

Select b = 10 m ; Circular pitch $p = \pi m$

$$y_2 = 0.154 - \frac{0.912}{Z_2} = 0.154 - \frac{0.912}{\frac{d_2}{m}} = 0.154 - \frac{0.912}{256}m = 0.154 - 3.5625\times10^{-3}\text{ m}$$

Mean pitch line velocity of weaker member $v_m = \dfrac{\pi d_2 n_2}{60{,}000} = \dfrac{\pi\times256\times250}{60000}$

= 3.351 m/sec

$\therefore$ Velocity factor $K_v = C_v = \dfrac{3}{3+v_m} = \dfrac{3}{3+3.351} = 0.4724$ ($\because v_m < 7.5 \dfrac{m}{Sec}$)

.... 2.128 (Old DDHB); 23.134 a (New DDHB)

Substituting all the values in Lewis equation

$$8057.81 = (379)(10m)(0.154 - 3.5625\times10^{-3}m)(\pi m)(0.4724)$$

$$1.4326 = 0.154m^2 - 3.5625\times10^{-3}m^3$$

$$\text{ie., } m^2 - 0.02313\,m^3 \geq 9.3$$

Trail : 1

Select m = 3 mm [Select standard module from Table 2.3 (Old DDHB) Table 23.3 (New DDHB)]

$8.3755 < 9.3$

$\therefore$ Not suitable

Trail : 2

Select m = 4 mm

$14.52 > 9.3$. Hence suitable

$\therefore$ Module m = 4mm

(c) Check for the stress

$$\text{Allowance stress } \sigma_{all} = (\sigma_{o2} K_v)_{all} = 379 \times 0.4724 = 179.0396 \text{ N/mm}^2$$

$$\text{Induced stress } \sigma_{ind} = (\sigma_{o2} K_v)_{ind} = \frac{F_{t_2}}{by_2p} = \frac{8057.81}{(10\times 4)(0.154 - 3.5625\times 10^{-3}\times 4)(\pi\times 4)}$$

$$= 114.708 \text{ N/mm}^2$$

Since $(\sigma_{o2} K_v)_{ind} < (\sigma_{o2} K_v)_{all}$, the design is safe

$\therefore$ module m = 4 mm

(iii) Dimensions

module m = 4 mm

Face width b = 10 m = 10 × 4 = 40 mm

$$\text{Number of teeth on pinion } z = \frac{d_1}{m} = \frac{64}{4} = 16$$

$$\text{Number of teeth on gear } z = \frac{d_2}{m} = \frac{256}{4} = 64$$

Centre distance a = 160 mm

From Table 23.1 (New DDHB) ; Table 2.1 (Old DDHB) for 20° full depth involute system

Addendum h_a = 1 m = 1 × 4 = 4 mm

Dedendum h_f = 1.25 m = 1.25 × 4 = 5 mm

Working depth h' = 2m = 2 × 4 = 8 mm

Total depth h = 2.25 m = 2.25 × 4 = 9 mm

$$\text{Tooth thickness } s = \frac{\pi m}{2} = \frac{\pi\times 4}{2} = 6.2832 \text{ mm}$$

Minimum clearance c = 0.25 m = 0.25 × 4 = 1 mm

Pitch circle diameter of pinion d_1 = 256 mm

Pitch circle diameter of gear d_2 = 64 mm

Outside or Addendum circle diameter of pinion $da_1 = (z_1+2)m = (16 + 2)\,4 = 72$ mm

Outside or Dedendum circle diameter gear $da_2 = (z_2 + 2)\,m = (64 + 2)\,4 = 264$ mm

Root or Dedendum circle diameter of pinion $d_{f_1} = d_1 - 2h_f$ .... 23.16 (New DDHB)

$$= 64 - 2 \times 5 = 54 \text{ mm}$$

Root or Dedendum circle diameter of gear $d_{f_2} = d_2 - 2h_f = 256 - 2 \times 5 = 246$ mm

Tangential tooth load F_t = 8057.81 N

Mean pitch line velocity v_m = 3.351 m/sec

Velocity factor K_v = $C_v = 0.4724$

Service factor C_s = 1.5

Circular pitch P = $\pi m = \pi \times 4 = 12.56.64$ mm

(iv) Checking

a) Dynamic load $$F_d = F_t + \frac{21 v_m (F_t + bc)}{21 v_m + \sqrt{F_t + bc}}$$

Tangential tooth load F_t = $F_{t_2} = 8057.81$ N

Mean pitch line velocity v_m = 3.351 m/sec

Face width b = 40 mm

From Fig. 2.29 (Old DDHB) ; Fig. 23.34a (New DDHB) for Class - III Precision gears when m = 4mm; Error f = 0.0125 mm.

From Table 2.35 (Old DDHB) : Table 23.32 (New DDHB) for $\alpha = 20°$ Full depth, Steel – Steel combination and f = 0.0215 mm

Dynamic load factor C = 145 kN/m = 145 N/mm

$$\therefore F_d = 8057.81 + \frac{21 \times 3.351\ (8057.81 + 40 \times 145)}{21 \times 3.351 + \sqrt{8057.81 + 40 \times 145}} = 13242.5 \text{ N}$$

(b) Wear load F_w = $d_1 bQK$ .... 2.160 (Old DDHB) ; 23.160 (New DDHB)

Ratio factor $Q = \dfrac{2z_2}{z_1 + z_2} = \dfrac{2 \times 64}{16 + 64} = 1.6$

For safer design $F_w \geq F_d$

ie., $d_1 bQK \geq F_d$

$64 \times 40 \times 1.6 \times K \geq 13242.5$

ie., $K \geq 3.233$ N/mm^2

From Table 2.40 (Old DDHB) ; Table 23.37B (New DDHB) for α = 20FD and $K \geq 3.233$ N/mm^2

Surface hardness for pinion = 450 BHN

Surface hardness for gear = 450 BHN

Example 4.11

A compressor running at 400 rpm is driven by a 25 kW, 1200 rpm motor through a pair of 14½° involute spur gear. The centre distance is around 400 mm. The pinion is made of forged steel of static allowable stress 190 MN/m^2 and 350 BHN. Gear is to be made of cast steel of static allowable stress 180 MN/mm^2 and 300 BHN. Design the gear for safe continuous operation. Check the gear for endurance, wear and dynamic strength. Name the class of the gear.

Data :

n_2 = 400 rpm; N = 25 kW; n_1 = 1200 rpm; $\alpha = 14\frac{1}{2}°$; a = 350 mm;
σ_{01} = 190 MN/m² = 190 N/mm²; σ_{02} = 180 MN/m² = 300 N/mm²
Surface hardness of pinion = 350 BHN
Surface hardness of gear = 300 BHN

Solution :

Velocity ratio $i = \frac{n_1}{n_2} = \frac{1200}{400} = 3$; Also $i = \frac{z_2}{z_1} = \frac{d_2}{d_1}$

Centre distance $a = \frac{d_1 + d_2}{2} = \frac{d_1 + id_1}{2} = \frac{(1+i)d_1}{2}$; i.e., $400 = \frac{(1+3)d_1}{2}$

Pitch circle diameter of pinion $d_1 = 200$ mm

$\therefore$ Pitch circle diameter of gear $d_2 = id_1 = 3 \times 200 = 600$ mm

To identify the weaker member temporarily assume $z_1 = 20$

$\therefore$ $z_2 = iz_1 = 3 \times 20 = 60$

Lewis form factor for 14½° involute $y = 0.124 - \frac{0.684}{z}$

Lewis form factor for pinion $y_1 = 0.124 - \frac{0.684}{z_1} = 0.124 - \frac{0.684}{20} = 0.0898$

Lewis form factor for gear $y_2 = 0.124 - \frac{0.684}{z_2} = 0.124 - \frac{0.684}{60} = 0.1126$

i) Identify the weaker member

Particulars	σ_0 N/mm²	y	$\sigma_0 y$	Remarks
Pinion	190	0.0898	17.062	Weaker
Gear	180	0.1126	20.268	

Since $\sigma_{01} y_1 < \sigma_{02} y_2$, pinion is weaker member. Therefore design should be based on pinion.

ii) Design

a) Tangential tooth load $F_t = \frac{9550 \times 1000 \times NC_s}{nr} = \frac{9550 \times 1000 \times PC_s}{nr}$ **where r in mm**

$\therefore$ Tangential tooth load of the weaker member $F_{t_1} = \frac{9550 \times 1000 NC_s}{n_1 r_1} = \frac{9550 \times 1000 PC_s}{n_1 r_1}$

From Table 14.4 (Old DDHB.Volume-I) or Table 14.7 (New DDHB) for the driven mechanism compressor load factor i.e., service factor C_s = 1.75

Pitch circle radius of pinion $r_1 = \frac{d_1}{2} = \frac{200}{2} = 100$ mm

$$\text{i.e. } F_{t_1} = \frac{9550\times1000\times25\times1.75}{1200\times100} = 3481.8 \text{ N}$$

b) Tangential tooth load from Lewis equation $F_t = \sigma_o bypC_v = \sigma_o bypK_v$ ---- 23.93 (New DDHB)

$\therefore$ Tangential tooth load of the weaker member $F_{t_1} = \sigma_{01} by_1pC_v = \sigma_{01} by_1pK_v$

Face width $b = 3\pi m$ to $4\pi m$ or $9.5\text{ m} < b < 12.5\text{ m}$

Select $b = 10\text{ m}$; circular pitch $p = \pi m$

$$y_1 = 0.124 - \frac{0.684}{z_1} = 0.124 - \frac{0.684}{\frac{d_1}{m}} = 0.124 - \frac{0.684m}{200} = 0.124 - 3.42\times10^{-3}m$$

---- **23.115 (New DDHB)**

Mean pitch line velocity of weaker member $v_m = \dfrac{\pi d_1 n_1}{60000} = \dfrac{\pi\times200\times1200}{60000} = 12.5664$ m/sec.

$\therefore$ Velocity factor $K_v = C_v = \dfrac{6}{6+v_m} = \dfrac{6}{6+12.5664} = 0.3232$ ($\because v_m < 20$ m/sec)

---- 2.130(Old DDHB); 23.136a (New DDHB)

Substituting all the values in Lewis equation

$$3481.8 = (190)(10m)(0.124 - 3.42\times10^{-3}m)(\pi m)(0.3232)$$

$$1.8048 = 0.124m^2 - 3.42\times10^{-3}m^3$$

i.e., $m^2 - 0.02758m^3 - 14.555 = 0$

i.e., $m^2 - 0.02758m^3 \geq 14.555$

Trail : 1

Select module $m = 4$ mm [Select standard module from Table 2.3 (Old DDHB); Table 23.3 (New DDHB)]

i.e., $4^2 - 0.02758\times4^3 \geq 14.555$

$14.235 < 14.555$

$\therefore$ Not suitable

Trail : 2

Select module $m = 5$ mm

i.e., $5^2 - 0.02758\times5^3 \geq 14.555$

$21.5525 > 14.555$

Hence suitable

$\therefore$ Module $m = 5$ mm

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{01} K_v)_{all} = 190\times0.3232 = 61.408$ N/mm²

Induced stress $\sigma_{ind} = (\sigma_{01} K_v)_{ind} = \dfrac{F_{t_1}}{by_1p} = \dfrac{3481.8}{(10\times5)(0.124 - 3.42\times10^{-3}\times5)(\pi\times5)} = 41.47$ N/mm²

---- 2.93(Old DDHB); 23.93 (New DDHB)

Since $(\sigma_{01} K_v)_{ind} < (\sigma_{01} K_v)_{all}$, the design is safe. Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth.

$$\text{Effective force } F_{eff} = \frac{F_{t_1}.C_s}{K_v} = \frac{F_{t_1}}{K_v} \quad \text{since } C_s \text{ is already considered}$$

$$= \frac{3481.8}{0.3232} = 10772.9\,N$$

Beam strength of weaker member $F_{b_1} = \sigma_{01}\, b y_1 p$

$$= (190)(10 \times 5)(0.124 - 3.42 \times 10^{-3} \times 5)(\pi \times 5) = 15952.2\ N$$

$$F_{b_2} > F_{eff}$$

$$\therefore\ FOS = \frac{F_{b_2}}{F_{eff}} = \frac{15952.2}{10772.9} = 1.5$$

The design is satisfactory and hence the module should be equal to 5 mm

$\therefore$ Module $m = 5$ mm.

iii) Dimensions

Module $m = 5$ mm; Face width $b = 10\,m = 10 \times 5 = 50$ mm

Number of teeth on pinion $z_1 = \frac{d_1}{m} = \frac{200}{5} = 40$

Number of teeth on gear $z_2 = \frac{d_2}{m} = \frac{600}{5} = 120$

Circular pitch $p = \pi m = \pi \times 5 = 15.708$ mm

Tangential tooth load $F_t = 3481.8$ N

Velocity factor $K_v = C_v = 0.3268$

Service factor $C_s = 1.75$

Mean pitch line velocity $v_m = 12.5664$ m/sec

Base circle diameter of pinion $d_{b_1} = d_1 \cos\alpha = 200 \cos 14½ = 193.63$ mm

Base circle diameter of gear $d_{b_2} = d_2 \cos\alpha = 600 \cos 14½ = 580.89$ mm

Pitch circle diameter of pinion $d_1 = 200$ mm

Pitch circle diameter of gear $d_2 = 600$ mm

From Table 2.1 (Old DDHB); Table 23.1 (New DDHB) for 14½° involute system

Addendum $h_a = 1\,m = 1 \times 5 = 5$ mm

Dedendum $h_f = 1.157\,m = 1.157 \times 5 = 5.785$ mm

Working depth $h' = 2\,m = 2 \times 5 = 10$ mm

Minimum total depth $h = 2.15\,m = 2.15 \times 5 = 10.75$ mm

Outside diameter (Addendum diameter) of pinion $da_1 = (z_1 + 2)\,m = (40 + 2)\,5 = 210$ mm

Outside diameter of gear $d_{a_2} = (z_2 + 2)\,m = (120 + 2)\,5 = 610$ mm

Tooth thickness $s = \frac{\pi}{2} m = \frac{\pi}{2} \times 5 = 0.7854$ mm

Minimum clearance $c = 0.157\,m = 0.157 \times 5 = 0.785$ mm

Dedendum circle diameter of pinion $d_{f_1} = d_1 - 2h_f = 200 - 2 \times 5.785 = 188.43$ mm

Dedendum circle diameter of gear $d_{f_2} = d_2 - 2h_f = 600 - 2 \times 5.785 = 588.43$ mm

iv) Checking

a) Dynamic load $F_d = F_t + \dfrac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}}$ ---- **2.148 a (Old DDHB); 23.155 (New DDHB)**

From Fig. 2.30 (Old DDHB); Fig. 23.35a (New DDHB), for $v_m = 12.5664$ m/sec

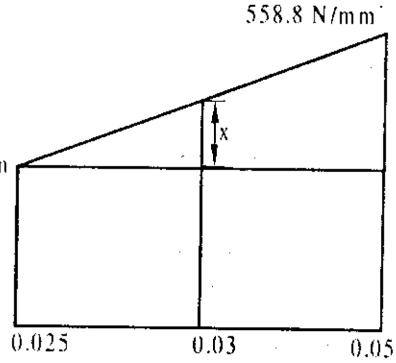
Error f = 0.03 mm

From Table 2.35 (Old DDHB); Table 23.32 (New DDHB) for $\alpha = 14½°$ and steel – steel combination

When error f = 0.025 mm ; C = 279.48 KN/m = 279.48 N/mm

When error f = 0.05 mm ; C = 558.8 KN/m = 558.8 N/mm

By interpolation



$$\frac{x}{558.8 - 279.48} = \frac{0.03 - 0.025}{0.05 - 0.025}$$

$\therefore$ x = 55.864 N/mm

$\therefore$ For error f = 0.03 mm;

Dynamic factor C = 279.48 + 55.864 = 335.344 N/mm

i.e. $F_d = 3481.8 + \dfrac{21 \times 12.5664[3481.8 + 50 \times 335.344]}{21 \times 12.5664 + \sqrt{3481.8 + 50 \times 335.344}}$

= 16637.1 N

b) Endurance strength $F_f F_{-1} = \sigma_{-1}\, b\, Ym = \sigma_{sf}\, b\, Ym$ ---- **2.153(Old DDHB); 23.158 (New DDHB)**

where $Y = \pi y_1 = \pi[0.124 - 3.42 \times 10^{-3} \times 5] = 0.336$

From Table 2.39 (Old DDHB); Table 23.33 (New DDHB) for surface hardness = 350 BHN

Select endurance limit $\sigma_{sf} = \sigma_{-1} = 620.5$ N/mm²

$\therefore F_f = F_{-1}$ = (620.5) (50) (0.336) (5) = 52122 N

As $F_f = F_{-1} > F_d$ the design will be satisfactory from the standpoint of strength.

c) Wear load $F_w = d_1 bQK$

Ratio factor $Q = \dfrac{2z_2}{z_1 + z_2} = \dfrac{2 \times 120}{40 + 120} = 1.5$

Average surface hardness for the gear pair $H_B = \dfrac{350 + 300}{2} = 325$ BHN

Surface fatigue stress $\sigma_{fac} = \sigma_{-1C} = (2.75\, H_B - 69)$ MPa ---- 2.168 c (Old DDHB); 23.168 (New DDHB)

= (2.75 × 325 – 69) = 824.75 N/mm²

From Table 2.8 [Old DDHB Volume I] or Table 2.10 (New DDHB Vol-I)

For steel Young's modulus $E_1 = E_2 = 206$ GPa $= 206 \times 10^3$ N/mm²

Equivalent Young's modulus $E_o = \dfrac{2E_1E_2}{E_1 + E_2} = \dfrac{2 \times 206 \times 10^3 \times 206 \times 10^3}{206 \times 10^3 + 206 \times 10^3} = 206 \times 10^3$ N/mm²

$\therefore$ Load stress factor $K = \dfrac{1.43(\sigma_{fac})^2 \sin\alpha}{E_o}$ ---- 2.161(Old DDHB); 23.161 (New DDHB)

$$= \frac{1.43 \times (824.75)^2 \times \sin 14\frac{1}{2}}{206 \times 10^3} = 1.18226$$

$\therefore$ Wear load $F_w = (200)(50)(1.5)(1.18226) = 17733.9$ N

As $F_w > F_d$, the design will be satisfactory from the stand point of wear or durability.

v) Class of gear

From Fig. 2.29 (Old DDHB); Fig 23.34a (New DDHB) for module m = 5 mm and error f = 0.03 mm

The class of the gear is "First class commercial gear".

Note : If $F_w < F_d$, then for safer design $F_w \geq F_d$ ie., d, b Q k $\geq F_d$

$\therefore$ k $\geq$ - - - N/mm²

Suggest suitable hardness for the gear pair using Table 23.37B (New DDHB) for the calculated value of K.

Example 4.12

Following are the details of a pair of spur gears

Details	**Pinion**	**Gear**
Rotational speed	**1440 rev/min**	**720 rev/min**
Allowable bending stress	**200 MPa**	**180 MPa**
Surface hardness	**250 BHN**	**200 BHN**
Modulus of elasticity	**200 GPa**	**200 GPa**
Tooth profile	**20° Full depth**	**20° full depth**
Centre distance	**132 mm**	
Face width	**32 mm**	

The pinion has 22 teeth. Determine the power that can be transmitted based on
(a) Bending strength (b) Surface endurance strength.
Gears are manufactured to have error in action less than 0.02 mm. What should be the endurance limit in bending of the weaker one to have endurance strength of 1.25 times the dynamic load.

VTU July/August 2004

Data :

$n_1 = 1440$ rpm; $n_2 = 720$ rpm; $\sigma_{01} = 200$ MPa; $\sigma_{02} = 180$ MPa;
$E_1 = E_2 = 200$ GPa $= 200 \times 10^3$ N/mm²; $\alpha = 20°$ FD; a = 132 mm; b = 32 mm;
$z_1 = 22$; error f < 0.02 mm; $F_{-1} = 1.25\, F_d$

Solution :

Velocity ratio $i = \frac{n_1}{n_2} = \frac{1440}{720} = 2$

Also $i = \frac{z_1}{z_2} = \frac{d_2}{d_1}$

$\therefore$ Number of teeth on gear $z_2 = iz_1 = 2 \times 22 = 44$

Centre distance $a = \frac{d_1 + d_2}{2} = \frac{d_1 + id_1}{2} = \frac{d_1(1+i)}{2}$

i.e., $132 = \frac{d_1(1+2)}{2}$

∴ Pitch circle diameter of pinion $d_1 = 88$ mm

Pitch circle diameter of gear $d_2 = id_1 = 2 \times 88 = 176$ mm

Also $d_1 = mz_1$

i.e., $88 = m \times 22$

∴ Module $m = 4$ mm

Lewis form factor for 20° full depth involute $y = 0.154 - \frac{0.912}{z}$ ---- 23.116 (New DDHB)

∴ Form factor for pinion $y_1 = 0.154 - \frac{0.912}{z_1} = 0.154 - \frac{0.912}{22} = 0.11255$

Form factor for gear $y_2 = 0.154 - \frac{0.912}{z_2} = 0.154 - \frac{0.912}{44} = 0.133273$

i) Identify the weaker member

Particulars	σ_o N/mm^2	y	$\sigma_o y$	Remarks
Pinion	200	0.11255	22.51	Weaker
Gear	180	0.133273	23.989	

Since $\sigma_{01} y_1 < \sigma_{02} y_2$, pinion is weaker. Hence design should be based on pinion.

a) i) Power transmitted based on bending strength

Tangential tooth load from Lewis equation $F_t = \sigma_o byp\, C_v = \sigma_o byp\, K_v$ ---- 2.93(Old DDHB); 23.93 (New DDHB)

Tangential tooth load of the weaker member $F_{t_1} = \sigma_{01} by_1 p\, C_v = \sigma_{01} by_1 p\, K_v$

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 88 \times 1440}{60000} = 6.635$ m/sec

∴ Velocity factor $K_v = C_v = \frac{3}{3 + v_m} = \frac{3}{3 + 6.635} = 0.3114$ since $v_m < 7.5$ m/sec

---- 2.128(Old DDHB); 23.134a (New DDHB)

$$\therefore F_{t_1} = (200)(32)(0.11255)(\pi \times 4)(0.3114) = 2818.4\text{ N}$$

$$\therefore \text{Also } F_{t_1} = \frac{9550 \times 1000 \times N}{n_1 r_1} = \frac{9550 \times 1000 \times P}{n_1 r_1}$$

$$\text{i.e., } 2818.4 = \frac{9550 \times 1000 \times P}{1440 \times \left(\frac{88}{2}\right)}$$

∴ Power transmitted based on bending strength, according to Lewis equation P = N = 18.7 kW. This power should by multiplied by C_s to get the design power

Power transmitted based on bending strength according to Spott's equation

$(F_{eff})(FOS) = F_b$ where F_b = Beam strength $= \sigma_b bmY$ and FOS = factor of safety.

Beam strength of the weaker member $F_b = (200)(32)(4)(\pi \times 0.11255) = 9051.8$

Assume FOS = 2

$$\therefore\ F_{eff} = \frac{9051.8}{2} = 4525.9\ \text{N}$$

Also $F_{eff} = C_s F_{t_1} + F_d$; Assume service factor $C_s = 1.5$

Assume both pinion and gear materials are steel $\therefore\ F_d = \dfrac{fn_1 z_1 b r_1 r_2}{2530\sqrt{r_1^2 + r_2^2}}$

Since error f < 0.02, take f = 0.015

Pitch circle radius of pinion $r_1 = \dfrac{d_1}{2} = \dfrac{88}{2} = 44$ mm

Pitch circle radius of gear $r_2 = \dfrac{d_2}{2} = \dfrac{176}{2} = 88$ mm

$$\therefore\ F_d = \frac{(0.015)(1440)(22)(32)(44)(88)}{2530\sqrt{44^2 + 88^2}} = 236.54\ \text{N}$$

$$\text{i.e., } 4525.9 = 1.5F_{t_1} + 236.54$$

$$\therefore\ F_{t_1} = 2859.6\ \text{N}$$

$$\text{Also } F_{t_1} = \frac{9550 \times 1000 \times N}{n_1 r_1} = \frac{9550 \times 1000 \times P}{n_1 r_1}$$

$$\text{i.e., } 2859.6 = \frac{9550 \times 1000 \times P}{1440 \times 44}$$

$\therefore$ Power transmitted based on bending strength according to Spott's equation P = N = 18.972 kW.

b) Power transmitted based on surface endurance strength

Wear load $F_w = d_1 bQK$

$$\text{Ratio factor } Q = \frac{2z_2}{z_1 + z_2} = \frac{2 \times 44}{22 + 44} = 1.333$$

Surface fatigue stress = $\sigma_{fac} = \sigma_{-1C} = 2.75\ H_B - 69 = 2.75 \times 200 - 69 = 481$ N/mm^2---- 23.168 (New DDHB)

$$\text{Equivalent Young's modulus } E_o = \frac{2E_1E_2}{E_1 + E_2} = \frac{2 \times 200 \times 10^3 \times 200 \times 10^3}{200 \times 10^3 + 200 \times 10^3} = 200 \times 10^3\ \text{N/mm}^2$$

$$\text{Load stress factor } K = \frac{1.43 \times 481^2 \times \sin 20}{200 \times 10^3} = 0.56578\ \text{N/mm}^2$$

$\therefore$ Wear load = (88) (32) (1.333) (0.56578) = 2123.784 N.

Considering service factor and factor of safety as unity, according to Spott's equation $F_w = F_{t_1} + F_d$

$$\text{where } F_d = \frac{fn_1 z_1 b r_1 r_2}{2530\sqrt{r_1^2 + r_2^2}} = 236.54\ \text{N}$$

i.e., $2123.784 = F_{t_1} + 236.54$ $\quad\therefore\ F_{t_1} = 1887.244$ N

$$\text{Also } F_{t_1} = \frac{9550 \times 1000 \times N}{n_1 r_1} = \frac{9550 \times 1000 \times P}{n_1 r_1}$$

$$\text{i.e.,}\quad 1887.244 = \frac{9550\times1000\times P}{1440\times44}$$

$\therefore$ Power transmitted based on wear according to Spott's equation $P = 12.52$ kW

c) Endurance limit of the weaker member according to Buckingham's equation

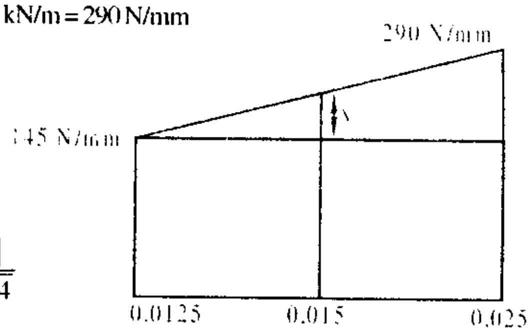
$$\text{Dynamic load } F_d = F_t + \frac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}} \quad \text{---- 2.148 a (Old DDHB); 23.155 (New DDHB)}$$

From Table 2.35 for 20° full depth, steel to steel

when $f = 0.0125$ mm ; $C = 145$ kN/m $= 145$ N/mm

when $f = 0.025$ mm ; $C = 290$ kN/m $= 290$ N/mm

By interpolation



$$\frac{x}{0.015-0.0125} = \frac{290-145}{0.025-0.0125}$$

$\therefore\ x = 29$ N/m

For error $f = 0.015$ mm, $C = 145 + 29 = 174$ N/mm

$$\therefore\ F_d = 2818.4 + \frac{21\times6.635[2818.4+32\times174]}{21\times6.635+\sqrt{2818.4+32\times174}}$$

$= 7878.85$ N

Endurance strength $F_r = F_{-1} = 1.25\,F_d = 1.25 \times 7878.85 = 9848.56$ N (given)

Also endurance strength $F_r = F_{-1} = \sigma_{-1}\,bYm = \sigma_{sf}\,bYm$---- 2.153(Old DDHB); 23.158 (New DDHB)

i.e., $9848.56 = (\sigma_{sf})(32)(\pi\times0.11255)(4)$ $(\because\ Y = \pi y_1)$

$\therefore$ Endurance limit $\sigma_{sf} = \sigma_{-1} = 217.6$ N/mm^2

Example 4.13

Design a spur gear drive to transmit 12 kW from 1500 rpm motor to a compressor run at 50 rpm

Data :

$N = 12$kW; $n_1 = 1500$ rpm; $n_2 = 50$ rpm

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{1500}{50} = 30$$

As the velocity ratio is more than 10, multistage reduction is recommended. Factorising 30 into 5 × 6, a two stage reduction drive as shown in Fig. 4.7 can be used.

Select the smaller speed pair for the gear design,

$$\text{i.e.,}\quad i = 6 = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$\therefore$ Speed of pinion $n_1 = 300$ rpm

Speed of gear $n_2 = 50$ rpm

Assume $\alpha = 20°$ full depth involute system

As the centre distance or diameter of gears or number of teeth on the gears are not given, refering Table 2.6 (Old DDHB); Table 23.6 (New DDHB) the minimum number of teeth on pinion to avoid interference for 20° full depth involute tooth is 17.

∴ Select number of teeth on pinion $z_1 = 18$

Number of teeth on gear $z_2 = iz_1 = 6 \times 18 = 108$

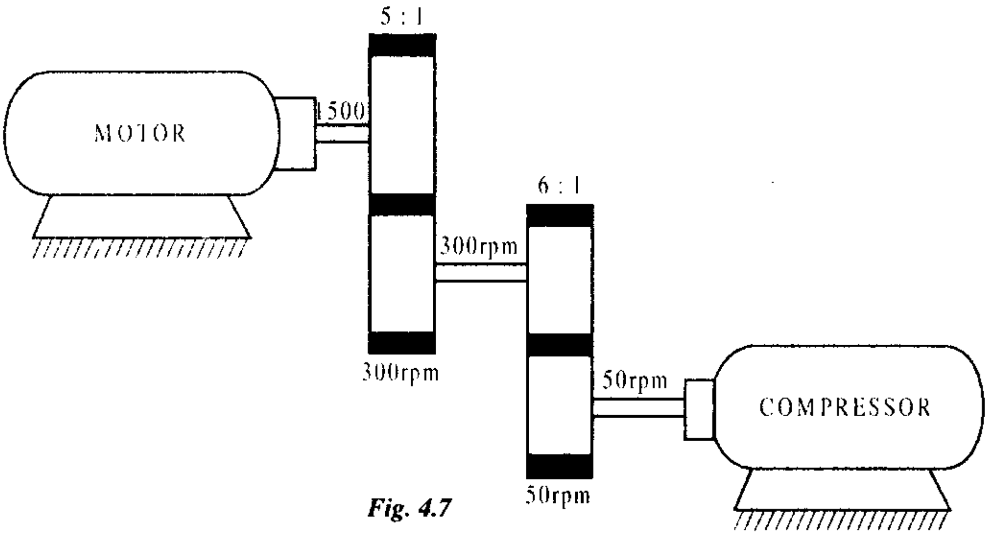


Fig. 4.7

Assume cast steel 0.20% C heat treated for pinion.

From Table 23.18 (New DDHB) for cast steel 0.20% C heat treated steel

Allowable static stress $\sigma_{01} = 173$ MPa = 173 N/mm^2

Lewis form factor for 20° FD involute $y = 0.154 - \frac{0.912}{z}$ ---- 2.98(Old DDHB); 23.116 (New DDHB)

$$\therefore \quad y_1 = 0.154 - \frac{0.912}{z_1} = 0.154 - \frac{0.912}{18} = 0.1033$$

$$y_2 = 0.154 - \frac{0.912}{z_2} = 0.154 - \frac{0.912}{108} = 0.1456$$

Equating $\sigma_{01}\, y_1$ to $\sigma_{02}\, y_2$

i.e., $\sigma_{01}\, y_1 = \sigma_{02}\, y_2$

$173 \times 0.1033 = \sigma_{02} \times 0.1456$

∴ $\sigma_{02} = 122.74$ N/mm^2

From Table 23.18 (New DDHB) the nearest value to 122.74 N/mm^2 is 124 N/mm^2

∴ Select SAE 1020 case hardened and WQT as gear material

Hence $\sigma_{02} = 124$ N/mm^2

Assume medium shock and 8-10 hours duty per day

∴ From Table 2.33 (Old DDHB); Table 23.13 (New DDHB) service factor $C_s = 1.5$

i) Identify the weaker member

Particulars	σ_o N/mm^2	y	$\sigma_o y$	Remarks
Pinion	173	0.1033	17.8709	Weaker
Gear	124	0.1456	18.0544	

Since $\sigma_{01}\, y_1 < \sigma_{02}\, y_2$, pinion is weaker. Therefore design should be based on pinion.

Design, dimensions, dynamic load, wear load surface hardness number, etc are as explained in Example 4.10.

Note :

According to Prof. MF Spotts

In order to avoid failure of gear tooth due to bending in the initial stages of design,

$$F_b > F_{eff}$$

Introducing factor of safety $F_b = F_{eff}\,(FOS)$ ---- (i)

Where F_b = Beam strength = $\sigma_b\, mbY$

F_{eff} = Effective load = $\dfrac{F_t C_s}{C_v}$ ---- (ii)

FOS = Factor of safety

σ_b = Allowable bending stress = $\dfrac{\sigma_u}{3}$ (weaker member)

σ_u = Ultimate strength

$Y = \pi y$

y = Weaker member form factor

m = Module

b = Face width

C_s = Service factor

C_v = Velocity factor

$F_t = \dfrac{9550 \times 1000N}{nr}$

r = Pitch circle radius of weaker member in mm

n = Speed of weaker member in rpm

N = Power in kW

The module obtained from equation (i) is based on beam strength.

Considering dynamic load according to Spotts $F_{eff} = C_s\, F_t + F_d$ ---- (iii)

Where F_d = Dynamic load = $\frac{en_1 z_1 b r_1 r_2}{2530\sqrt{r_1^2 + r_2^2}}$ for steel pinion and steel gear

$F_d = \frac{en_1 z_1 b r_1 r_2}{3785\sqrt{r_1^2 + r_2^2}}$ for CI gear and CI pinion

$F_d = \frac{en_1 z_1 b r_1 r_2}{3260\sqrt{r_1^2 + r_2^2}}$ for steel pinion and CI gear

Where e = Sum of error between two meshing teeth in mm = f

= $e_g + e_p$ = error for gear + error for pinion

n_1 = Speed of pinion

z_1 = Number of teeth on pinion

b = Face width

r_1 = Pitch circle radius of pinion

r_2 = Pitch circle radius of gear

Considering the dynamic load to avoid failure of the gear tooth due to bending

$F_b = F_{eff}$ (FOS)

$= (F_t C_s + F_d)$ (FOS) ---- (iv)

In order to avoid the failure of the gear tooth due to pitting

$F_w > F_{eff}$

Considering factor of safety

$F_w = (F_{eff})$ (FOS) Where F_w = Wear load = $d_1 bQK$

d_1 = Pitch circle diameter of pinion

b = Face width

Q = Ratio factor = $\frac{2z_2}{z_1 + z_2}$

K = Load stress factor = $\frac{1.43\sigma_{-1C}^2 \sin\alpha}{E_o}$

E_o = Equivalent Young's modulus = $\frac{2E_1 E_2}{E_1 + E_2}$

E_1 = Young's modulus for pinion material

E_2 = Young's modulus for gear material

α = Pressure angle

σ_{-1C} = Limiting surface fatigue stress = $(2.75\ H_B - 69)$ MPa

H_B = Hardness Number (BHN) of the weaker member

$$F_{eff} = C_s F_t + F_d \text{ considering dynamic load}$$

$$F_{eff} = \frac{C_s F_t}{C_v} \text{ Neglecting dynamic load.}$$

∴ Considering dynamic load to avoid failure of gear due to pitting

$$F_w = (C_s F_t + F_d) \text{ FOS} \quad \text{---- (v)}$$

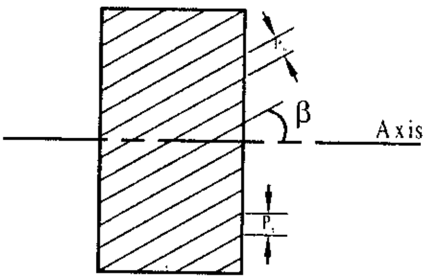
Considering the FOS as unity $F_w = C_s F_t + F_d$

The gear can also be designed using Prof. MF Spott's equations.

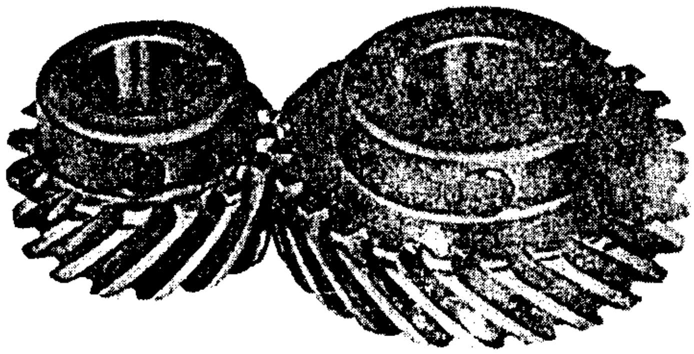
4.15 HELICAL GEARS

Helical gears are gears in which the teeth are cut in the form of helix around the gear. Helical gears are used to connect parallel shafts and also non parallel, non intersecting shafts. Helical gears used to transmit power between parallel shafts are called parallel helical gears, and helical gears used to transmit power between non parallel shafts are called crossed helical gears. Helical gear teeth pickup the load gradually. This gradual contact across the teeth results in less impact loading and thus helical gears operate more quietly than spur gears, have longer life and are stronger. Meshing helical gears must have the same helix angle but opposite hand of helix. As a result of helix angle, both radial and thrust loads are imposed on the helical gear support bearings.

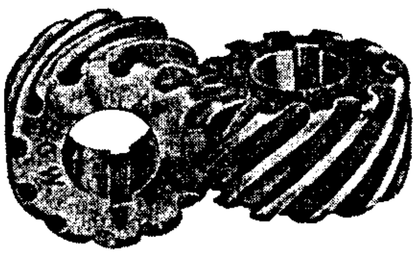
Fig : 4.8 a shows a helical gear. Fig. 4.8 b shows a parallel helical gear. Fig. 4.8 c shows a crossed helical gear.



(a)



(b)



(c)

Fig. 4.8

i) **Helix angle (β) :** It is the angle between a element of the helical tooth and the axis of rotation of the gear.

ii) **Transverse Circular pitch (p_t) :** It is the distance between corresponding points on adjacent tooth measured on the pitch circle.

iii) **Normal circular pitch :** It is the distance between corresponding points on adjacent tooth measured in a direction normal to the tooth.

$$p_n = p_t \cos \beta$$

Normal diametral pitch

$$p_{nd} = \frac{\pi}{p_n} = \frac{p_d}{\cos\beta}$$

$$p_d = \text{diametral pitch} = \frac{\pi\cos\beta}{p_n} = \frac{\pi\cos\beta}{\pi m_n} = \frac{\cos\beta}{m_n}$$

$$\text{Also } p_d = \frac{z_1}{d_1} = \frac{z_2}{d_2}; \quad \therefore z_1 = p_d d_1 = \frac{d_1 \cos\beta}{m_n}$$

$$\text{i.e., } d_1 = \frac{m_n z_1}{\cos\beta}$$ ---- 2.227(Old DDHB); 23.227 (New DDHB)

$$\text{similarly } z_2 = p_d d_2 = \frac{d_2 \cos\beta}{m_n} \quad \therefore d_2 = \frac{m_n z_2}{\cos\beta}$$ ---- 2.228(Old); 23.228 (New DDHB)

$$\text{Centre distance } c = \frac{d_1 + d_2}{2}$$ ---- 2.229(Old DDHB); 23.229 (New DDHB)

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$ ---- 2.210(Old DDHB); 23.210m (New DDHB)

Where β = Helix angle

m_n = Normal module

d_1 = pitch circle diameter of pinion

d_2 = pitch circle diameter of gear

4.16 DOUBLE HELICAL GEAR

A double helical gear is as shown in Fig. 4.9. These gears have two sets of opposed helical teeth. One having a right hand helix and the other left hand helix. These two sets of teeth are often cut on the same gear blank with a small space separating them. Axial thrust which occurs in case of single helical gears is eliminated in double helical gears. It is used to transmit heavy loads at high speeds.

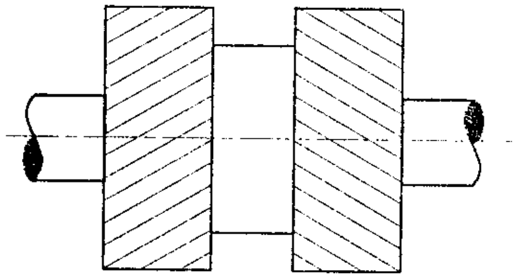
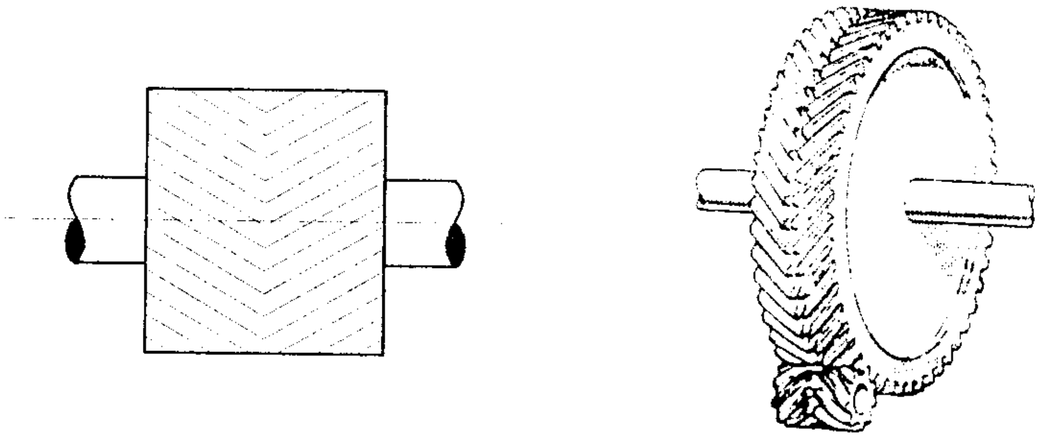


Fig. 4.9

4.16.1 HERRINGBONE GEAR

It is basically the same as the double helical gears but in this gear there is no space separating the two opposed sets of helical teeth as shown in Fig. 4.10 a and Fig. 4.10 b.



(a) *(b)*

Fig. 4.10

4.17 Helical gear forces

The resultant force F acting on the tooth of a helical gear is resolved into three components F_t, F_r and F_a as shown in Fig. 4.11 a.

Let F_t = tangential force = $\frac{M_{t1}}{r_1} = \frac{M_{t2}}{r_2}$

F_r = separating force or radial force

F_a = axial or thrust force

F = resultant force

β = Helix angle

α_n = normal pressure angle = pressure angle measured in a plane perpendicular to tooth

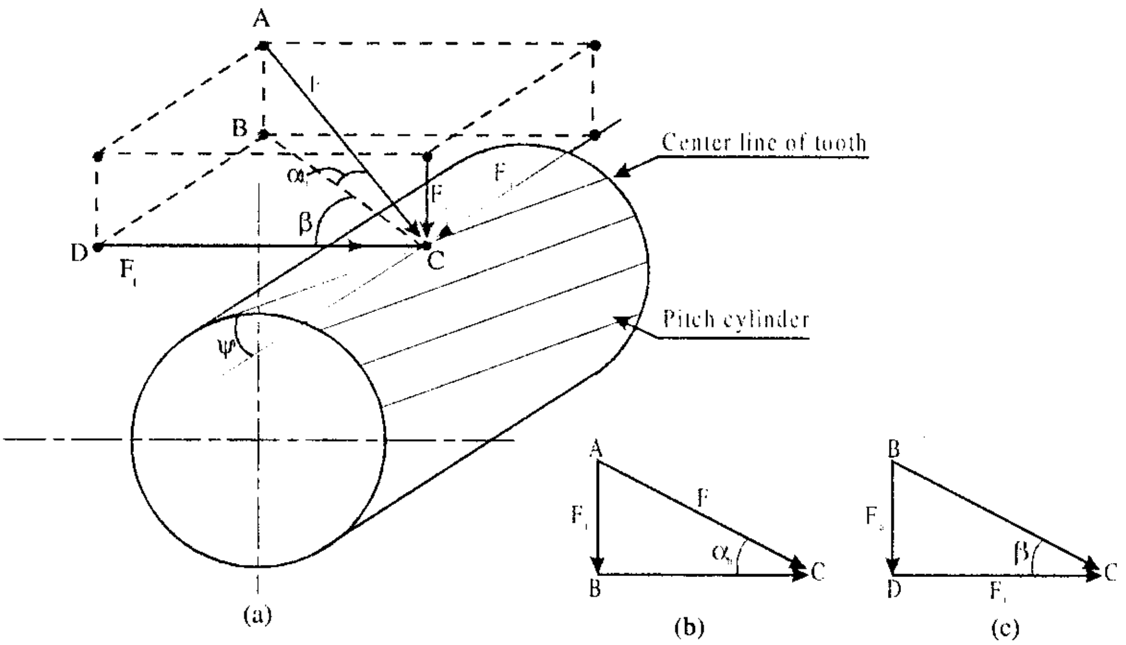


Fig. 4.11

M_t = torque on the gear

r = Pitch circle radius

α_t = Pressure angle measured perpendicular to the axis of gear

From Δ^{le} ABC (Fig. 4.11 b)

$$\sin\alpha_n = \frac{F_r}{F} \qquad \therefore\ F_r = F\sin\alpha_n \qquad \text{---- (i)}$$

$$\cos\alpha_n = \frac{BC}{F} \qquad \therefore\ BC = F\cos\alpha_n \qquad \text{---- (ii)}$$

From Δ^{le} BDC (Fig. 4.11 c)

$$\cos\beta = \frac{F_t}{BC} = \frac{F_t}{F\cos\alpha_n} \qquad \therefore\ F_t = F\cos\alpha_n\cos\beta \qquad \text{---- (iii)}$$

$$\sin\beta = \frac{F_a}{BC} = \frac{F_a}{F\cos\alpha_n} \qquad \therefore\ F_a = F\cos\alpha_n\sin\beta \qquad \text{---- (iv)}$$

Equation (iv) divided by (iii) gives

$$\frac{F_a}{F_t} = \frac{F\cos\alpha_n\sin\beta}{F\cos\alpha_n\cos\beta} = \tan\beta$$

$\therefore$ Axial force $F_a = F_t\tan\beta$ ---- (v)

Equation (i) divided by (iii) gives

$$\frac{F_r}{F_t} = \frac{F\sin\alpha_n}{F\cos\alpha_n\cos\beta} = \frac{\tan\alpha_n}{\cos\beta}$$

$\therefore$ Radial force $F_r = F_t\left(\frac{\tan\alpha_n}{\cos\beta}\right)$ ----(vi)

From (iii) Tangential force $F_t = F\cos\alpha_n\cos\beta$

The direction of the thrust or axial component depends upon the hand of the helix and the direction of rotation of the gear under consideration.

The following points should be remembered while determining the direction of thrust component.

(i) Select the driving gear.

(ii) Use right hand for RH helix and left hand for LH helix.

(iii) Keep the fingers in the direction of rotation of the driver gear and the thumb will indicate the direction of the thrust component for the driving gear.

(iv) The direction of the thrust component for the driven gear is opposite to that of the driver gear.

4.18 VIRTUAL OR FORMATIVE OR EQUIVALENT NUMBER OF TEETH

The number of teeth of the equivalent spur gear in the normal plane is known as formative number of teeth or equivalent number of teeth or virtual number of teeth.

The pitch cylinder of the helical gear is cut by plane A-A, which is normal to the tooth elements as shown in Fig. 4.12. The intersection of the plane A-A and the pitch cylinder produces an ellipse. The semi-major and semi-minor axes of this ellipse are $\left(\frac{d}{2\cos\beta}\right)$ and $\left(\frac{d}{2}\right)$ respectively. It can be proved from analytical geometry that the radius of curvature r_c at point B is given by

i.e., radius of curvature of ellipse $r_c = \frac{a^2}{b}$

where a and b are semi-major and semi-minor axes respectively. Substituting the values of a and b in the expression for r_c,

$$r_c = \frac{d}{2\cos^2\beta} \quad \text{---- (i)}$$

In the design of helical gears, an imaginary spur gear is considered in plane A-A

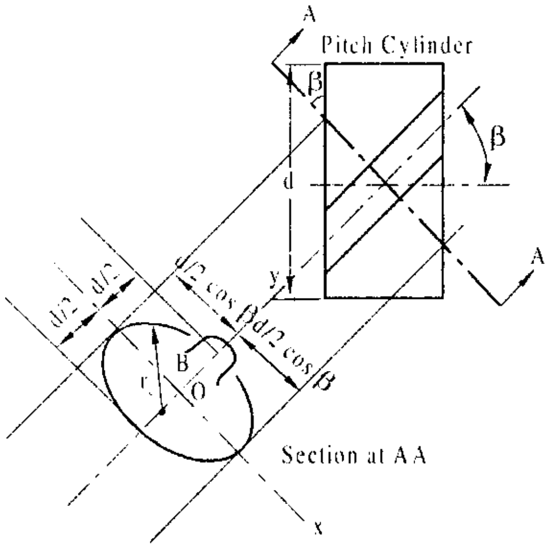


Fig. 4.12

with a pitch circle radius of r_c and module m_n. It is called a 'formative' or 'virtual' spur gear. The pitch circle diameter d_c of the virtual gear is given by

$$d_c = \frac{d}{\cos^2\beta} \qquad \text{---- (ii)}$$

The number of teeth z_v on this imaginary spur gear are the virtual number of teeth. It is given by

$$z_v = \frac{2\pi r_c}{p_n} = \frac{2\pi(d/2\cos^2\beta)}{(\pi m_n)} = \frac{d}{m_n\cos^2\beta} = \frac{m_n z}{\cos\beta}\cdot\frac{1}{m_n\cos^2\beta} \quad \left(\because\ d = \frac{m_n z}{\cos\beta}\right)$$

$$\therefore \text{ Formative number of teeth } z_v = \frac{z}{\cos^3\beta} \qquad \text{---- (iii)}$$

where z is the actual number of teeth and β = Helix angle

The equivalent number of teeth is used to determine the Lewis form factors y

$$y = 0.124 - \frac{0.684}{z_v} \text{ for } 14\tfrac{1}{2}^\circ \text{ involute}$$ ----2.97(Old DDHB); **23.115 (New DDHB)**

$$y = 0.154 - \frac{0.912}{z_v} \text{ for } 20^\circ \text{ full depth involute}$$ ----2.98(Old DDHB); **23.116 (New DDHB)**

$$y = 0.17 - \frac{0.95}{z_v} \text{ for } 20^\circ \text{ stub involute}$$ ----2.99(Old DDHB); **23.117 (New DDHB)**

4.19 BEAM STRENGTH OF HELICAL GEARS

In order to determine beam strength, the helical gear is considered to be equivalent to a formative spur gear.

The formative gear is an imaginary spur gear in a plane perpendicular to the tooth element. The pitch circle diameter of this gear is d_c, the number of teeth is z_v and the module m_n. The beam strength of the spur gear is given by,

$F_b = mb\,\sigma_b Y$ ---- (i)

This equation is applicable to the formative spur gear.

Referring to Fig. 4.13

$F_b = (F_b)_n$ = beam strength perpendicular to the tooth element;

$m = m_n$ = normal module; face width $b = \dfrac{b}{\cos\beta}$

and $Y = \pi y$

y = Lewis form factor based on virtual number of teeth z_v. Substituting these values in equation (i)

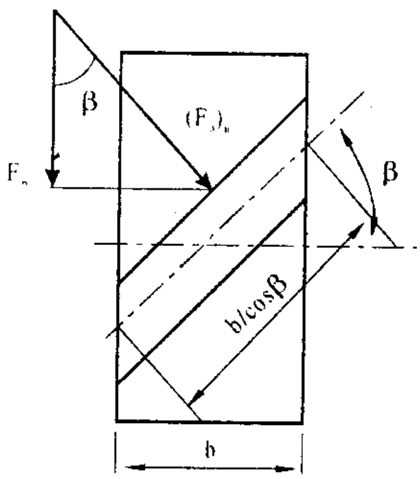


Fig. 4.13

$$(F_b)_n = \frac{m_n b \sigma_b Y}{\cos\beta} \quad \text{---- (ii)}$$

In Fig. 4.13 F_b is the component of $(F_b)_n$ in the plane of rotation. Thus,

$$F_b = (F_b)_n \cos\beta \quad \text{---- (iii)}$$

From (ii) and (iii)

$$F_b = m_n b \sigma_b Y \quad \text{---- (iv)}$$

Equation (iv) is known as Lewis equation for helical gears. In this equation, the form factor Y is based on the virtual number of teeth. The beam strength F_b indicates the maximum value of tangential force that the tooth can transmit without bending failure. It should be always more than the effective force between the meshing teeth.

The modified form of lewis equation used in the design of helical gear is,

$$F_t = \frac{\sigma_0 b y p_t C_v \cos\beta}{C_w} = \frac{\sigma_0 b y p_n C_v}{C_w} = \frac{\sigma_0 b y p_n K_v}{C_w} \quad \text{since } p_t = \frac{p_n}{\cos\beta} \quad \text{---- 2.286(Old)}$$

; 23.286 (New DDHB)

where $K_v = C_v$ = Velocity factor

C_w = Wear and lubrication factor ---- Table 2.56(Old DDHB); **Table 23.47 (New DDHB)**

σ_0 = Static allowable stress ---- Table 2.57(Old DDHB); **Table 23.48 (New DDHB)**

$p_n = \pi m_n$ = normal pitch, m_n = normal module

4.20 DYNAMIC LOAD

The dynamic load is assumed to be inclined at an angle in to the tangential plane. Referring to Fig. 4.11a, the dynamic load F_d acts in the same direction as the resultant force F. According to Buckingham, the dynamic load of the helical gear is given by the equation.

$$F_d = F_t + \frac{21v_m(F_t + bC\cos^2\beta)\cos\beta}{21v_m + \sqrt{F_t + bC\cos^2\beta}}$$ ---- 2.297 a(Old DDHB): **23.309a (New DDHB)**

4.21 ENDURANCE STRENGTH

The endurance strength of the helical gear is given by

$F_f = F_{-1} = \sigma_{-1}\, bY\, m_t\cos\beta = \sigma_f\, bY\, m_t\cos\beta$ ---- 2.290(Old DDHB); **23.293 (New DDHB)**

$$= \sigma_{-1}\, b\,(\pi y)\,\frac{m_n}{\cos\beta}\cos\beta = \sigma_f\, b\,(\pi y)\,\frac{m_n}{\cos\beta}\cos\beta$$

$$= \sigma_{-1}\, bp_n y = \sigma_f\, bp_n y \quad [\because\ \pi m_n = p_n]$$

The dynamic load should be less than the endurance strength by a reasonable margin of safety.

4.21.1 WEAR LOAD

The wear strength indicates the maximum tangential force that the tooth can transmit without pitting failure. It should be always more than the effective force between the meshing teeth. According to Buckingham's equation

Wear load $F_w = \dfrac{d_1 bQK}{\cos^2\beta}$ ---- 2.298(Old DDHB); **23.310 (New DDHB)**

Ratio factor $Q = \dfrac{2z_{2v}}{z_{1v} + z_{2v}} = \dfrac{2z_2}{z_1 + z_2}$

Load stress factor $K = \dfrac{\sigma_{-1c}^2 \sin\alpha_n}{0.7E_o} = \dfrac{\sigma_{fc}^2 \sin\alpha_n}{0.7E_o}$ ---- 2.299(Old DDHB); **23.311 (New DDHB)**

where $\tan\alpha_n = \tan\alpha_t \cos\beta$ and α_n = Normal pressure angle

Equivalent young's modulus $E_o = \dfrac{2E_1E_2}{E_1 + E_2}$

Limiting stress for surface fatigue $\sigma_{fac} = \sigma_{-1C} = (2.75\, H_B - 69)$ MPa

---- 2.291 c(Old DDHB); **23.294a (New DDHB)**

For safer design wear load must be greater than dynamic load.

4.21.2 Procedural steps for the design of helical gear

The design procedure for helical gear is almost the same as that of spur gear

Let m_n = Normal module

m_t = Transverse module = $\frac{m_n}{\cos\beta}$

p_n = Normal pitch

p_t = Transverse pitch = $\frac{p_n}{\cos\beta}$

α_n = Normal pressure angle

i) Identify the weaker member

It is first necessary to determine which is weaker, the gear or pinion

Particulars	σ_o	y	σ_o y	Remarks
Pinion	σ_{01}	y_1	σ_{01} y_1	
Gear	σ_{02}	y_2	σ_{02} y_2	

From Table 2.57 (Old DDHB); **Table 23.48 (New DDHB)** obtain the allowable static stresses σ_{01} and σ_{02} for the given materials. For the given normal pressure angle and tooth form, Lewis form factor y for pinion and gear can be obtained using the following formulae.

$$y = 0.124 - \frac{0.684}{z_v} \text{ for } 14\tfrac{1}{2}^\circ \text{ involute}$$ ---- 2.97(Old DDHB); **23.115 (New DDHB)**

$$y = 0.154 - \frac{0.912}{z_v} \text{ for } 20^\circ \text{ full depth involute}$$ ---- 2.98(Old DDHB); **23.116 (New DDHB)**

$$y = 0.17 - \frac{0.95}{z_v} \text{ for } 20^\circ \text{ stub involute}$$ ---- 2.99(Old DDHB); **23.117 (New DDHB)**

where z_v = Virtual number of teeth = $\frac{z}{\cos^3\beta}$ ---- 2.285(Old DDHB); **23.285 (New DDHB)**

The gear whose value of $\sigma_o y$ is less is the weaker member

i.e., if $\sigma_{01}\, y_1 < \sigma_{02}\, y_2$, pinion is weaker

if $\sigma_{02}\, y_2 < \sigma_{01}\, y_1$, Gear is weaker

Design should be based on weaker member

ii) Design based on module in normal plane (m_n) as standard module

a) Tangential tooth load

$$F_t = \frac{9550 \times 1000 \times NC_s}{nr} = \frac{9550 \times 1000 \times PC_s}{nr}$$ ---- 2.87 b(Old DDHB); 23.87b (New DDHB)

where r = Radius of the weaker member = $\frac{d}{2} = \frac{m_n z}{2\cos\beta}$

n = Speed of weaker member

C_S = Service factor ---- Table 2.33(Old DDHB); Table 23.13 (New DDHB)

b) Tangential tooth load from Lewis equation

$$F_t = \frac{\sigma_o b y p_t C_v \cos\beta}{C_w} = \frac{\sigma_o b y p_t K_v \cos\beta}{C_w}$$ ---- 2.286(Old DDHB); 23.286 (New DDHB)

$$= \frac{\sigma_o b y p_n K_v}{C_w} \quad \left(\because p_t = \frac{p_n}{\cos\beta}\right)$$

$$\frac{p_t}{\tan\beta} < b < \frac{20m_t}{\tan\beta}$$ ---- 2.277 and 2.280(Old DDHB); 23.277and 23.280 (New DDHB)

i.e., $$\frac{p_n}{\cos\beta.\tan\beta} < b < \frac{20m_n}{\cos\beta.\tan\beta}$$

i.e., $$\frac{\pi m_n}{\sin\beta} < b < \frac{20m_n}{\sin\beta} \quad \therefore \text{ Select } b = 10\ m_n$$

C_w = wear and lubrication factor ---- Table 2.56(Old); **Table 23.47 (New DDHB)**

$$K_v = C_v = \text{velocity factor} = \frac{4.5}{4.5 + v_m} \text{ up to 12.5 m/sec}$$

---- 2.288(Old DDHB); 23.288a (New DDHB)

$$= \frac{6}{6 + v_m} \text{ up to 20 m/sec}$$ ---- 2.289 a(Old DDHB); 23.89a (New DDHB)

$$= \frac{5.6}{5.6 + \sqrt{v_m}} = \frac{5.55}{5.55 + \sqrt{v_m}} \text{ over 20 m/sec}$$ ---- 2.289 b(Old); 23.290a (New DDHB)

By equating the equations obtained from (a) and (b) and by trial and error method, find module 'm_n'

(c) Check for the stress

Calculate the induced stress by the equation

$$\sigma_{ind} = (\sigma_o\ K_v)_{ind} = \frac{F_t.C_w}{b y p_n}$$ ---- 2.286(Old DDHB); 23.286 (New DDHB)

Allowable stress $\sigma_{all} = (\sigma_o\ K_v)_{all}$

If $(\sigma_o\ K_v)_{ind} < (\sigma_o\ K_v)_{all}$, then the design is satisfactory.

iii) Dimensions

Calculate all the important geometric parameters of tooth profile by using the equations given in table 2-1

i.e., Addendum h_a, Dedendum h_f, Tooth thickness, Total depth h, Clearance c, Outside diameter of pinion and gear (d_{a_1} and d_{a_2})

Also calculate, Tangential tooth load F_t using the equation from (ii) (a)

Center distance $a = \frac{d_1 + d_2}{2}$ ---- 2.229(Old DDHB); 23.229 (New DDHB)

Face width b

Root or dedendum circle diameter of pinion and gear using formulas 2.246 and 2.247(Old DDHB); 23.246 and **23.247 (New DDHB)**.

iii) Checking

a) Dynamic load

According to Buckingham's equation

Dynamic load $F_d = F_t + \frac{21v_m(F_t + bC\cos^2\beta)\cos\beta}{21v_m + \sqrt{F_t + bC\cos^2\beta}}$ ---- 2.297 a(Old); **23.309a (New DDHB)**

To find dynamic load factor C, use Tables 2.34 and 2.35(Old DDHB); **Fig. 23.35a and Table 23.32 (New DDHB)**

Use Fig. 2.29 (Old DDHB); Fig. 23.34a (New DDHB) to find the error 'f' if the class of gear is known.

(b) Endurance strength

$F_f = F_{-1} = \sigma_{-1} bYm_t\cos\beta = \sigma_f bYm_t\cos\beta$ ---- 2.290(Old DDHB); 23.293 (New DDHB)

$= (\sigma_{-1})(b)(\pi y)\frac{m_n}{\cos\beta}\cos\beta = \sigma_f\, b\,(\pi y)\frac{m_n}{\cos\beta}\cos\beta$

$= \sigma_{-1} bp_n y = \sigma_f bp_n y$

For safer design F_d must be less than the allowable endurance strength.

(c) Wear Load

According to Buckingham's equation

Wear load $F_w = \frac{d_1 bQK}{\cos^2\beta}$ ---- 2.298(Old DDHB); **23.310 (New DDHB)**

Ratio factor $Q = \frac{2z_{2v}}{z_{1v} + z_{2v}} = \frac{2z_2}{z_1 + z_2}$

Load stress factor $K = \frac{\sigma_{-1C}^2 \sin\alpha_n}{0.7E_o} = \frac{\sigma_{fc}^2 \sin\alpha_n}{0.7E_o}$ ---- 2.299(Old DDHB); **23.311 (New DDHB)**

Equivalent young's modulus $E_o = \frac{2E_1E_2}{E_1 + E_2}$

Limiting stress for surface fatigue $\sigma_{fac} = \sigma_{-1C} = (2.75H_B - 69)$ MPa

---- 2.291 c(Old DDHB); **23.294a (New DDHB)**

For safer design F_w must be greater than F_d

If $F_w < F_d$ then make one or more following changes

(i) Calculate error F_d by decreasing error 'f' (ii) Decrease the module m_n

(iii) Increase the face width b (iv) Increase the surface hardness.

Example 4.14

A pair of mating helical gears have 20° pressure angle in the normal plane. The normal module is 5 mm and the module in the diametral plane is 5.7735 mm. The pitch diameter of the smaller gear is 115.47 mm. If the transmission ratio is 4:1 calculate (i) Helix angle (ii) Normal pitch (iii) Transverse pitch (iv) Number of teeth for each gear (v) Addendum (vi) Dedendum (vii) Whole depth (viii) Clearance (IX) Tooth thickness (x) Working depth (xi) Outside diameters (xii) Centre distance (xiii) Root or dedendum circle diameters (xiv) Base circle diameters.

Data :

$$\alpha_n = 20°;\ m_n = 5\text{ mm};\ m_t = 5.7735\text{ mm};\ d_1 = 115.47\text{ mm};\ i = 4$$

Solution :

i) Helix angle

Transverse module $m_t = \dfrac{m_n}{\cos\beta}$ ---- 2.212 (Old DDHB); 23.212(New DDHB)

i.e., $5.7735 = \dfrac{5}{\cos\beta}$

$\therefore$ Helix angle $\beta = 30°$

ii) Normal pitch

$p_n = \pi m_n = \pi \times 5 = 15.708$ mm ---- 2.211(Old DDHB); 23.211 (New DDHB)

iii) Transverse pitch

$p_t = \pi m_t = \pi \times 5.7735 = 18.138$ mm ---- 2.213(Old DDHB); 23.213 (New DDHB)

iv) Number of teeth on each gear

Velocity ratio $i = \dfrac{n_1}{n_2} = \dfrac{z_2}{z_1} = \dfrac{d_2}{d_1}$

Pitch circle diameter of pinion $d_1 = \dfrac{m_n z_1}{\cos\beta}$ ---- 2.227 (Old DDHB); 23.227 (New DDHB)

i.e., $115.47 = \dfrac{5 \times z_1}{\cos 30}$

$\therefore$ Number of teeth on pinion $z_1 = 20$

$\therefore$ Number of teeth on gear $z_2 = iz_1 = 4 \times 20 = 80$

From Table 2.1 (Old DDHB); **Table 23.1 (New DDHB)** for pressure angle $\alpha_n = 20°$ (full depth)

v) Addendum $h_a = 1\,m_n = 1 \times 5 = 5$ mm

vi) Dedendum $h_f = 1.25\,m_n = 1.25 \times 5 = 6.25$ mm

vii) Whole depth $h = 2.25\,m_n = 2.25 \times 5 = 11.25$ mm

viii) Clearance $c = 0.25\,m_n = 0.25 \times 5 = 1.25$ mm

ix) Tooth thickness $s = \frac{\pi}{2} m_n = \frac{\pi}{2} \times 5 = 7.854$ mm

x) Working depth $h' = 2 m_n = 2 \times 5 = 10$ mm

xi) Outside diameter of pinion $da_1 = d_1 + 2 h_a = 115.47 + 2 \times 5 = 125.47$ mm ---- 2.245(Old); 23.245 (New)

Outside diameter of gear $da_2 = d_2 + 2 h_a = 461.88 + 2 \times 5 = 471.88$ mm

---- 2.246(Old DDHB); 23.246 (New DDHB)

xii) Centre distance $a = \frac{d_1 + d_2}{2}$ ---- 2.229(Old DDHB); 23.229 (New DDHB)

Pitch circle diameter of gear $d_2 = id_1 = 4 \times 115.47 = 461.88$ mm

$\therefore$ Centre distance $a = \frac{115.47 + 461.88}{2} = 288.675$ mm

xiii) Root or dedendum circle diameter of pinion $d_{f_1} = d_1 - 2 h_f$ ---- 2.247(Old DDHB); 23.247 (New DDHB)

$= 115.47 - 2 \times 6.5 = 102.97$ mm

Root or dedendum circle diameter of gear $d_{f_2} = d_2 - 2 h_f$ ---- 2.248(Old DDHB); 23.248 (New DDHB)

$= 461.88 - 2 \times 6.5 = 449.38$ mm

xiv) Base circle diameter of pinion $d_{b_1} = d_1 \cos\alpha_t$ ---- 2.249(Old DDHB); 23.249 (New DDHB)

Transverse pressure angle, $\tan \alpha_t = \frac{\tan \alpha_n}{\cos\beta} = \frac{\tan 20}{\cos 30}$ ---- 2.221(Old DDHB); 23.221 (New DDHB)

$\therefore$ Transverse pressure angle, $\alpha_t = 22.796°$

$\therefore \; d_{b_1} = 115.47 \cos 22.796 = 106.45$ mm

Base circle diameter of gear $d_{b_2} = d_2 \cos \alpha_t = 461.88 \cos 22.796 = 425.8$ mm ---- 2.250(Old); 23.250 (New)

Example 4.15

A pair of parallel helical gears is shown in Fig. 4.14. A 5 kW power at 720 rpm is supplied to pinion through its shaft. The normal module is 5 mm and the normal pressure angle is 20°. The pinion has right hand helix, while the gear has left hand helix. The helix angle is 30°. The arrow indicates the direction of rotation when seen from the right hand side. Determine the components of the tooth force and draw a free body diagram showing the forces acting on the pinion and the gear.

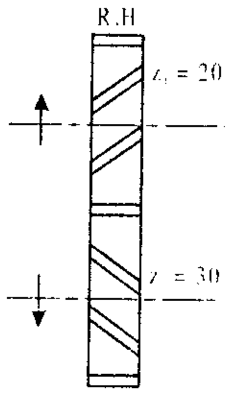


Fig. 4.14

Data :

$P = N = 5kW_s\ n_1 = 720$ rpms; $m_n = 5$ mm; $\alpha_n = 20°$; $\beta = 30°$; $z_1 = 20$; $z_2 = 30$

Solution :

Torque on the pinion shaft $M_{t_1} = \dfrac{60 \times 10^6 \times N}{2\pi n_1} = \dfrac{60 \times 10^6 \times 5}{2\pi \times 720} = 66314.56\,\text{Nmm}$

Pitch circle radius of pinion $r_1 = \dfrac{m_n z_1}{2\cos\beta} = \dfrac{5 \times 20}{2\cos 30} = 57.735\,\text{mm}$

Pitch circle radius of gear $r_2 = \dfrac{m_n z_2}{2\cos\beta} = \dfrac{5 \times 30}{2\cos 30} = 86.6\,\text{mm}$

Tangential force $F_t = \dfrac{M_{t_1}}{r_1} = \dfrac{66314.56}{57.735} = 1148.6\,\text{N}$

Separating or Radial force $F_r = \dfrac{F_t \tan\alpha_n}{\cos\beta} = \dfrac{1148.6 \times \tan 20}{\cos 30} = 482.73\,\text{N}$

Axial or thrust force $F_a = F_t \tan\beta = 1148.6 \times \tan 30 = 663.145\,\text{N}$

The free body diagram of forces acting on the pinion and gear is shown in Fig. 4.15.

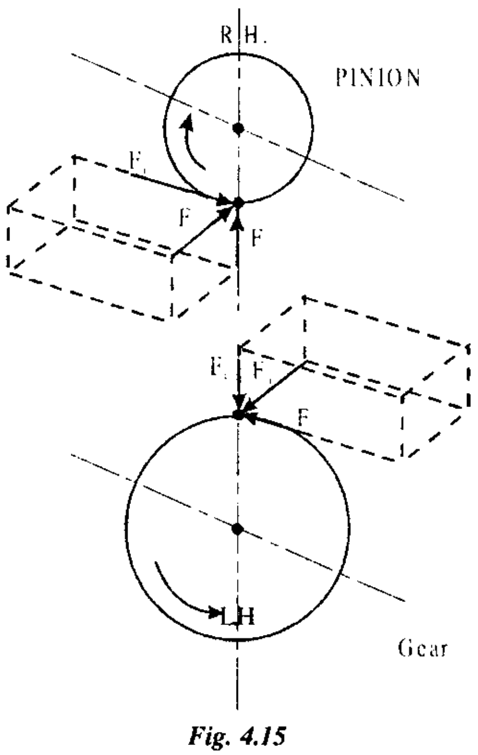


Fig. 4.15

Example 4.16

Design a pair of helical gears to transmit power of 15 kW at 3200 rpm with speed reduction 4:1 Pinion is made of cast steel 0.4 % C untreated. Gear made of high grade CI. Helix angle is limited to 26° and not less than 20 teeth are to be used on either gear. Suggest suitable surface hardness for the gear pair.

Data :

$$P = N = 15\ kW;\ n_1 = 3200\ rpm;\ \beta = 26°;\ z_1 = 20;\ i = 4;$$

Pinion material – 0.4% C untreated; Gear - High grade CI

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\therefore \text{ Speed of gear } n_2 = \frac{n_1}{i} = \frac{3200}{4} = 800\ rpm$$

Number of teeth on gear $z_2 = iz_1 = 4 \times 20 = 80$

From Table 2.57 (Old DDHB): Table 23.48 (New DDHB) allowable stress for 0.4 % C untreated

$$\sigma_{01} = 69.6\ MPa = 69.6\ N/mm^2$$

High grade CI $\sigma_{02} = 31\ MPa = 31\ N/mm^2$

Formative number of teeth $z_v = \dfrac{z}{\cos^3\beta}$ ---- 2.285(Old DDHB); 23.285 (New DDHB)

$$\therefore \text{ Formative number of teeth on pinion } z_{1v} = \frac{z_1}{\cos^3\beta} = \frac{20}{\cos^3 26} = 27.545$$

$$\text{Formative number of teeth on gear } z_{2v} = \frac{z_2}{\cos^3\beta} = \frac{80}{\cos^3 26} = 110.18$$

Assume pressure angle in the normal plane $\alpha_n = 20°$ full depth

Lewis form factor for 20° full depth $y = 0.154 - \dfrac{0.912}{z_v}$ ---- 2.98(Old DDHB); 23.116 (New DDHB)

$$\therefore \text{ Form factor for pinion } y_1 = 0.154 - \frac{0.912}{z_{1v}} = 0.154 - \frac{0.912}{27.545} = 0.1208$$

$$\text{Form factor for gear } y_2 = 0.154 - \frac{0.912}{z_{2v}} = 0.154 - \frac{0.912}{110.18} = 0.1457$$

i) Identify the weaker member

Particulars	σ_0 N/mm²	y	$\sigma_o y$	Remarks
Pinion	69.6	0.1208	8.4077	
Gear	31	0.1457	4.516	Weaker

As $\sigma_{02} y_2 < \sigma_{01} y_1$, gear is weaker. Therefore design should be based on gear.

ii) Design

a) Tangential tooth load

$$F_t = \frac{9550\times1000NC_s}{nr} = \frac{9550\times1000PC_s}{nr} \quad \text{where r is mm}$$

---- 23.87 b (New DDHB)

$\therefore$ Tangential tooth load of the weaker member gear $F_{t_2} = \frac{9550\times1000NC_s}{n_2r_2} = \frac{9550\times1000PC_s}{n_2r_2}$

Pitch circle radius of gear $r_2 = \frac{d_2}{2} = \frac{m_n z_2}{2\cos\beta} = \frac{m_n\times80}{2\cos26} = 44.5\ m_n$

Assume medium shock and 8 – 10 hours duty per day

$\therefore$ From Table 2.33 (Old DDHB); Table 23.13 (New DDHB), service factor $C_s = 1.5$

$$\text{i.e., } F_{t_2} = \frac{9550\times1000\times15\times1.5}{800\times44.5m_n} = \frac{6035.8}{m_n} \qquad \text{---- (i)}$$

b) Lewis equation for tangential tooth load

$$F_t = \frac{\sigma_o byp_tC_v\cos\beta}{C_w} = \frac{\sigma_o byp_nC_v}{C_w} = \frac{\sigma_o byp_nK_v}{C_w} \quad (\because p_t = \frac{p_n}{\cos\beta})$$

---- 2.286(Old DDHB); **23.286 (New DDHB)**

Since gear is the weaker member

$$F_{t_2} = \frac{\sigma_{02}by_2p_nC_v}{C_w} = \frac{\sigma_{02}by_2p_nK_v}{C_w}$$

Assume scant lubrication but frequent inspection

$\therefore$ From Table 2.56 (Old DDHB); **Table 23.47 (New DDHB)**, $C_w = 1.25$

$$\frac{p_t}{\tan\beta} < b < \frac{20m_t}{\tan\beta} \qquad \text{---- 2.277 and 2.280(Old DDHB); 23.277 and 23.280 (New DDHB)}$$

$$\text{i.e., } \frac{\pi m_n}{\sin\beta} < b < \frac{20m_n}{\sin\beta}$$

$\therefore$ Select $b = 10m_n$

$$\text{i.e., } F_{t_2} = \frac{(31)(10m_n)(0.1457)(\pi m_n)K_v}{1.25} = 113.517\,m_n^2K_v \qquad \text{---- (ii)}$$

Equating equations (i) and (ii)

$$113.517\,m_n^2K_v = \frac{6035.8}{m_n}$$

$$\therefore\ m_n^3K_v = 53.171 \qquad \text{---- (iii)}$$

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_2n_2}{60000} = \frac{\pi m_nz_2n_2}{60000\cos\beta} = \frac{\pi m_n 80\times800}{60000\times\cos26} = 3.7284\ m_n$ m/sec

Trial : 1

Select module $m_n = 5$ mm [select standard module from Table 2.3 (Old DDHB); Table 23.3 (New DDHB)]

$$\therefore \quad v_m = 3.7284 \times 5 = 18.642 \text{ m/sec}$$

Velocity factor $K_v = \frac{6}{6 + v_m}$ ($\because v_m < 20$ m/sec) ---- 2.289 a(Old DDHB); **23.289a (New DDHB)**

$$= \frac{6}{6 + 18.642} = 0.2435$$

Now from equation (iii)

$$(5^3)0.2435 \geq 53.171$$

$$30.4375 < 53.171$$

$\therefore$ Not suitable

Trial : 2

Select module $m_n = 6$mm

$$v_m = 3.7284 \times 6 = 22.3704 \text{ m/sec}$$

Velocity factor $K_v = C_v = \frac{5.6}{5.6 + \sqrt{v_m}}$ since $v_m > 20$ m/sec

---- 2.289 b(Old); 23.290a (New DDHB)

$$= \frac{5.6}{5.6 + \sqrt{22.3704}} = 0.542$$

From equation (iii)

$$(6^3)(0.542) \geq 53.71$$

$$117.1 > 53.71$$

Hence suitable

$\therefore$ Normal module $m_n = 6$ mm

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{02} K_v)_{all} = 31 \times 0.54 = 16.74$ N/mm^2

Induced stress $\sigma_{ind} = (\sigma_{02} K_v)_{ind} = \frac{F_{t2} . C_v}{b y_2 p_n}$ ---- 2.286(Old DDHB); 23.286 (New DDHB)

$$= \frac{\left(\frac{6035.8}{6}\right) \times 1.25}{(10 \times 6)(0.1457)(\pi \times 6)} = 7.63 \text{ N/mm}^2$$

Since $(\sigma_{02} K_v)_{ind} < (\sigma_{02} K_v)_{all}$ the design is safe.

Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth

Effective force $F_{eff} = \frac{F_{t2} C_s}{K_v} = \frac{F_{t2}}{K_v}$ since C_s is already considered

$$= \frac{(6035.8/6)}{0.542} = 1856.032 \text{ N}$$

Beam strength of weaker member $F_{b2} = \frac{\sigma_{02} b y_2 p_n}{C_w} = \frac{(31)(10\times6)(0.1457)(\pi\times6)}{1.25} = 4086.6\ N$

$$F_{b2} > F_{eff}$$

$$\therefore\ FOS = \frac{F_{b2}}{F_{eff}} = \frac{4086.6}{1856.032} = 2.2$$

The design is satisfactory and hence the module in normal plane should be equal to 6 mm

i.e., Normal module $m_n = 6mm$

iii) Dimensions

Module in normal plane $m_n = 6mm$

Module in diametral plane $m_t = \frac{m_n}{\cos\beta} = \frac{6}{\cos 26} = 6.6756\ mm$

Face width $b = 10\,m_n = 10\times6 = 60\ mm$

$$b_{min} = \frac{\pi m_n}{\sin\beta} = \frac{\pi\times6}{\sin 26} = 43\ mm$$

since $b > 43$ mm, safe

Normal pitch $P_n = \pi m_n = \pi\times6 = 18.85\ mm$

Axial pitch $p_t = \pi m_t = \pi\times6.6756 = 20.972\ mm$

Pitch circle diameter of pinion $d_1 = \frac{m_n z_1}{\cos\beta} = \frac{6\times20}{\cos 26} = 133.5\ mm$

Pitch circle diameter of gear $d_2 = \frac{m_n z_2}{\cos\beta} = \frac{6\times80}{\cos 26} = 534\ mm$

Centre distance $a = \frac{d_1+d_2}{2} = \frac{133.5+534}{2} = 333.75\ mm$

From Table 2.1 (Old DDHB); Table 23.1 (New DDHB) for pressure angle 20° full depth involute system

Addendum $h_a = 1\,m_n = 1\times6 = 6\ mm$

Dedendum $h_f = 1.25\,m_n = 1.25\times6 = 7.5\ mm$

Whole depth $h = 2.25\,m_n = 2.25\times6 = 13.5\ mm$

Clearance $c = 0.25\,m_n = 0.25\times6 = 1.5\ mm$

Tooth thickness $s = \frac{\pi}{2}m_n = \frac{\pi}{2}\times6 = 9.425\ mm$

Working depth $h' = 2\,m_n = 2\times6 = 12\ mm$

Outside or Addendum circle diameter of pinion $d_{a1} = d_1 + 2\,h_a$ ---- 2.245(Old); 23.245 (New DDHB)

$= 133.5 + 2\times6 = 145.5\ mm$

Addendum circle diameter of gear $d_{a2} = d_2 + 2\,h_a$ ---- 2.246(Old DDHB); 23.246 (New DDHB)

$= 534 + 2\times6 = 546\ mm$

Root or dedendum circle diameter of pinion $d_{f1} = d_1 - 2\,h_f$ ---- 2.247(Old DDHB); 23.247 (New DDHB)

$= 133.5 - 2\times7.5 = 118.5\ mm$

Root or dedendum circle diameter of gear $d_{f2} = d_2 - 2\,h_f$ ---- 2.248(Old DDHB); 23.248 (New DDHB)

$$= 534 - 2 \times 7.5 = 519 \text{ mm}$$

For transverse pressure angle $\tan\alpha_t = \dfrac{\tan\alpha_n}{\cos\beta}$ ---- 2.221(Old DDHB); 23.221 (New DDHB)

$$= \frac{\tan 20}{\cos 26} = 0.404954$$

$\therefore$ Pressure angle in the diametral of plane $\alpha_t = 22.0457°$

Base circle diameter of pinion $db_1 = d_1 \cos\alpha_t$ ---- 2.249(Old DDHB); 23.249 (New DDHB)

$$= 133.5 \cos 22.0457 = 123.74 \text{ mm}$$

Base circle diameter of gear $db_2 = d_2 \cos\alpha_t = 534 \cos 22.0457 = 494.96$ mm.

---- 2.250(Old); 23.250 (New DDHB)

Mean pitch line velocity $v_m = 22.3704$ m/sec

Velocity factor $K_v = C_v = 0.54$

Service factor $C_s = 1.5$

Wear and lubrication factor $C_w = 1.25$

Tangential tooth load $F_t = \dfrac{6035.8}{m_n} = \dfrac{6035.8}{6} = 1006$ N

iv) Checking

a) Dynamic Load

According to Buckingham's equation,

Dynamic load $F_d = F_t + \dfrac{21 v_m \left(F_t + bC\cos^2\beta\right)\cos\beta}{21 v_m + \sqrt{F_t + bC\cos^2\beta}}$ ---- 2.297 a(Old); 23.309a (New DDHB)

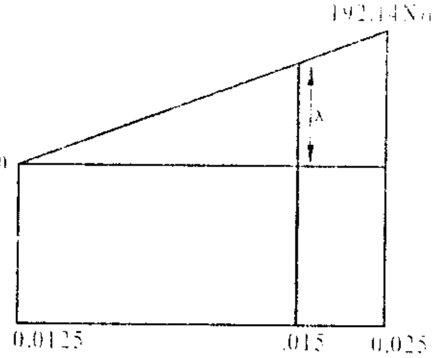
From Fig. 2.30 (Old DDHB); Fig. 23.35a (New DDHB) for $v_m = 22.3704$ m/sec

Error f = 0.015 mm

From Table 2.35 (Old DDHB); **Table 23.32 (New DDHB)** for 20° F.D. Steel - CI combination

when error f = 0.0125 mm C = 99.57 kN/m = 99.57 N/mm; when error f = 0.025 mm, c = 192.14 $\dfrac{KN}{M}$

By interpolation



$$\frac{x}{192.14 - 99.57} = \frac{0.015 - 0.0125}{0.025 - 0.0125}$$

$\therefore x = 18.514$ N/mm

$\therefore$ For error f = 0.015 mm

Dynamic factor C = 99.57 + 18.514 = 118.084 N/mm

$$F_d = 1006 + \frac{21 \times 22.3704\left[1006 + 60 \times 118.084\cos^2 26\right]\cos 26}{21 \times 22.3704 + \sqrt{1006 + 60 \times 118.084\cos^2 26}}$$

$$= 6155.27 \text{ N}$$

(b) Wear load

According to Buckingham's equation

$$\text{Wear load } F_w = \frac{d_1 b Q K}{\cos^2 \beta} \quad \text{---- 2.298(Old DDHB); \textbf{23.310 (New DDHB)}}$$

$$\text{Ratio factor } Q = \frac{2z_{2v}}{z_{1v} + z_{2v}} = \frac{2z_2}{z_1 + z_2} = \frac{2 \times 80}{20 + 80} = 1.6$$

For safer design $F_w \geq F_d$

$$\text{ie., } \frac{d_1 b Q K}{\cos^2 \beta} \geq F_d$$

$$\text{ie., } \frac{133.5 \times 60 \times 1.6 \times K}{\cos^2 26} \geq 6155.27$$

$$\text{ie., } K \geq 0.388 \text{ N/mm}^2$$

From Table 23.37B (New DDHB) for $\alpha = 20°$ and $K \geq 0.388$ N/mm²

Surface hardness for pinion = 200 BHN

Surface hardness for gear = 150 BHN

Example 4.17

A pair of carefully cut (Class - II) helical gears for a turbine has a transmission ratio of 10:1 the teeth are 20° stub involute in the normal plane. Pinion has 25 teeth and rotates at 5000 rpm. Material for pinion and gear is 0.4% carbon steel untreated. Determine the module in normal plane, diametral plane and face width of the gears. ***Suggest suitable hardness.*** **Modulus of elasticity may be taken as 210 Gpa. Helix angle = 30°. Power transmitted = 90 kW.**

Data :

$i = 10$; $n_1 = 5000$ rpm ; $\alpha_n = 20°$ stub; $z_1 = 25$; $E_1 = E_2 = 210$ GPa = 210×10^3 N/mm²;

$\beta = 30°$; $P = N = 90$ kW; Material for pinion and gear : 0.4% carbon steel untreated.

Solution :

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\text{Speed of gear } n_2 = \frac{n_1}{i} = \frac{5000}{10} = 500 \text{ rpm}$$

$$\text{Number of teeth on gear } z_2 = iz_1 = 10 \times 25 = 250$$

From Table 2.57 (Old DDHB); **Table 23.48 (New DDHB) for 0.4% carbon steel untreated**

$$\sigma_{b1} = \sigma_{b2} = 69.6 \text{ Mpa} = 69.6 \text{ N/mm}^2$$

$$\text{Formative number of teeth } z_v = \frac{z}{\cos^3 \beta} \quad \text{---- 2.285(Old DDHB); 23.285 (New DDHB)}$$

$\therefore$ Formative number of teeth on pinion $z_{1v} = \frac{z_1}{\cos^3\beta} = \frac{25}{\cos^3 30} = 38.49$

Formative number of teeth on gear $z_{2v} = \frac{z_2}{\cos^3\beta} = \frac{250}{\cos^3 30} = 384.9$

Lewis form factor for 20° stub $y = 0.17 - \frac{0.95}{z_v}$ ---- 2.99(Old DDHB); 23.117 (New DDHB)

Lewis form factor for pinion $y_1 = 0.17 - \frac{0.95}{z_{1v}} = 0.17 - \frac{0.95}{38.49} = 0.14532$

Lewis form factor for gear $y_2 = 0.17 - \frac{0.95}{z_{2v}} = 0.17 - \frac{0.95}{384.9} = 0.167532$

i) Identify the weaker member

As pinion and gears material are the same, pinion is weaker member. Therefore design should be based on pinion.

ii) Design

a) Tangential tooth load $F_t = \frac{9550 \times 1000 NC_S}{nr} = \frac{9550 \times 1000 PC_S}{nr}$ where r in mm

Tangential tooth load of the weaker member pinion $F_{t1} = \frac{9550 \times 1000 NC_S}{n_1 r_1} = \frac{9550 \times 1000 PC_S}{n_1 r_1}$

Pitch circle radius of pinion $r_1 = \frac{d_1}{2} = \frac{m_n z_1}{2\cos\beta} = \frac{m_n \times 25}{2\cos 30} = 14.4338\ m_n$

From Table 14.4 (Old DDHB - Vol - I) or Table 14.7 (New DDHB Vol-I) for turbine blower (driven by motor) Load factor i.e., service factor $C_s = 1.25$

$$\text{i.e., } F_t = \frac{9550 \times 1000 \times 90 \times 1.25}{5000 \times 14.4338\, m_n} = \frac{14886.98}{m_n} \quad \text{----(i)}$$

(b) Lewis equation for tangential tooth load

$$F_t = \frac{\sigma_o b y p_t C_v \cos\beta}{C_w} = \frac{\sigma_o b y p_n C_v}{C_w} \quad \left(\because p_t = \frac{p_n}{\cos\beta}\right)$$

$$= \frac{\sigma_o b y p_n K_v}{C_w} \quad \text{---- 2.286(Old DDHB); 23.286 (New DDHB)}$$

Since pinion is the weaker member

$$F_{t1} = \frac{\sigma_{01} b y_1 p_n C_v}{C_w} = \frac{\sigma_{01} b y_1 p_n K_v}{C_w}$$

Assume scant lubrication but frequent inspection

$\therefore$ From Table 2.56(Old DDHB); Table 23.47 (New DDHB) $C_w = 1.25$

$$\frac{p_t}{\tan\beta} < b < \frac{20m_t}{\tan\beta}$$ ---- 2.277 and 2.280(Old DDHB); 23.277 and 23.280 (New DDHB)

i.e., $\frac{\pi m_n}{\sin\beta} < b < \frac{20m_n}{\sin\beta}$

$\therefore$ Select $b = 10\ m_n$

$$\therefore\ F_u = \frac{(69.6)(10m_n)(0.14532)(\pi m_n)(K_v)}{1.25} = 254.2\ m_n^2 K_v \quad \text{---- (ii)}$$

Equating equations (i) and (ii)

$$254.2\ m_n^2 K_v = \frac{14886.98}{m_n}$$

$$\text{i.e., } m_n^3 K_v = 58.564 \quad \text{---- (iii)}$$

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_1 n_1}{60000}$

$$= \frac{\pi m_n z_1 n_1}{60000\cos\beta} = \frac{\pi m_n 25 \times 5000}{60000 \times \cos 30} = 7.5575\ m_n \text{ m/sec}$$

Trial : 1

Select module $m_n = 5$ mm (Select standard module from Table 2.3 (Old DDHB); Table23.3 (New DDHB))

$$\therefore\ v_m = 7.5575 \times 5 = 37.7875 \text{ m/sec}$$

Velocity factor $K_v = C_v = \frac{5.6}{5.6 + \sqrt{v_m}}$ Since $v_m > 20$ m/sec

---- 2.289 b(Old DDHB); 23.290a (New DDHB)

$$\text{Velocity factor } K_v = C_v = \frac{5.6}{5.6 + \sqrt{37.7875}} = 0.4767$$

Substituting in equation (iii)

$$(5^3)\ 0.4767 \geq 58.564$$

$$59.5875 > 58.564$$

Hence suitable

Trial : 2

Select module $m_n = 4$ mm

$$\therefore\ v_m = 7.5575 \times 4 = 30.23 \text{ m/sec}$$

$$\text{Velocity factor } K_v = C_v = \frac{5.6}{5.6 + \sqrt{v_m}} = \frac{5.6}{5.6 + \sqrt{30.23}} = 0.5046$$

From equation (iii) $(4^3)\ (0.5046) \geq 58.514$

$$32.2944 < 58.564$$

$\therefore$ Not suitable

$\therefore$ Module in the normal plane $m_n = 5$ mm.

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{01}K_v)_{all} = 69.6 \times 0.4767 = 33.178 \text{ N/mm}^2$

Induced stress $\sigma_{ind} = (\sigma_{01}K_v)_{ind} = \dfrac{F_{t1}.C_w}{by_1p_n}$ ---- 2.286(Old DDHB); 23.286 (New DDHB)

$$= \frac{\left(\frac{14886.98}{5}\right)(1.25)}{(10\times5)(0.14532)(\pi\times5)} = 32.61 \text{ N/mm}^2$$

Since $(\sigma_{01}C_v)_{ind} < (\sigma_{01}C_v)_{all}$, the design is safe. Also in order to avoid breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth.

Effective force $F_{eff} = \dfrac{F_{t1}C_s}{K_v} = \dfrac{F_{t1}}{K_v}$ since C_s is already considered

$$= \frac{\frac{14886.98}{5}}{0.4767} = 6245.85 \text{ N}$$

Beam strength of weaker member $F_{b1} = \dfrac{\sigma_{01}by_1p_n}{C_w}$

$$= \frac{(69.6)(10\times5)(0.14532)(\pi\times5)}{1.25} = 6354.98 \text{ N}$$

Since $F_{b1} > F_{eff}$, the design is satisfactory and hence the module in normal plane should be equal to 5 mm.

∴ Module in the normal plane $m_n = 5$ mm

Module in the diametral plane $m_t = \dfrac{m_n}{\cos\beta} = \dfrac{5}{\cos 30} = 5.7735$ mm

Face width $b = 10m_n = 10\times5 = 50$ mm

$$b_{min} = \frac{\pi m_n}{\sin\beta} = \frac{\pi\times5}{\sin 30} = 31.4 \text{ mm}$$

Since $b > 31.4$ mm, safe

∴ Face width $b = 50$mm.

iii) Checking

a) Dynamic load

According to Buckingham's equation,

Dynamic load $F_d = F_t + \dfrac{21v_m\left(F_t + bC\cos^2\beta\right)\cos\beta}{21v_m + \sqrt{F_t + bC\cos^2\beta}}$ ---- 2.297 a(Old DDHB); 23.309a (New DDHB)

From Fig. 2.29 (Old DDHB); Fig. 23.34a (New DDHB) for carefully cut gear when module = 5 mm

error $f = 0.025$ mm

From Table 2.35 (Old DDHB); Table 23.32 (New DDHB) for 20° stub, steel – steel combination and f = 0.025 mm

Dynamic load factor $C = 300.2$ kN/m $= 300.2$ N/mm

Tangential tooth load $F_t = \frac{14886.98}{5} = 2977.4$ N

Mean velocity $v_m = 37.7875$ m/sec.

i.e., $F_d = 2977.4 + \frac{21 \times 37.7875[2977.4 + 50 \times 300.2\cos^2 30]\cos 30}{21 \times 37.7875 + \sqrt{2977.4 + 50 \times 300.2\cos^2 30}}$

$= 13693.93$ N

(b) Wear load according to Buckingham's equation

$F_w = \frac{d_1 bQK}{\cos^2 \beta}$ ----2.298(Old DDHB); 23.310 (New DDHB)

d_1 = Pitch circle diameter of pinion $= \frac{m_n z_1}{\cos\beta} = \frac{5 \times 25}{\cos 30} = 144.338$ mm

Ratio factor $Q = \frac{2z_{2v}}{z_{1v} + z_{2v}} = \frac{2z_2}{z_1 + z_2} = \frac{2 \times 250}{25 + 250} = 1.8182$

Load stress factor $K = \frac{\sigma_{-1C}^2 \sin\alpha_n}{0.7E_o} = \frac{\sigma_{fc}^2 \sin\alpha_n}{0.7E_o}$ ---- 23.311 (New DDHB)

Equivalent Young's modulus $E_o = \frac{2E_1E_2}{E_1 + E_2} = \frac{2 \times 210 \times 10^3 \times 210 \times 10^3}{210 \times 10^3 + 210 \times 10^3} = 210 \times 10^3$ N/mm^2

For safer design

$F_w \geq F_d$

i.e., $\frac{d_1 bQK}{\cos^2 \beta} \geq F_d$

i.e., $\frac{(144.338)(50)(1.8182)(K)}{\cos^2 30} \geq 13693.93$N

$\therefore$ Load stress factor $K \geq 0.7827$ N/mm^2

i.e., $\frac{(\sigma_{fc})^2 \sin\alpha_n}{0.7E_o} \geq 0.7827$

i.e., $\frac{(\sigma_{fc})^2 \sin 20}{0.7 \times 210 \times 10^3} \geq 0.7827$

$\therefore$ Limiting stress for surface fatigue $\sigma_{fc} = \sigma_{-1C} \geq 580$ N/mm^2

From Table 2.40 (Old DDHB); Table 23.37B (New DDHB)for $\alpha = 20°$ $K \geq 0.7827$ N/mm^2 and

$\sigma_{fc} = 580$ N/mm^2

Surface hardness for pinion = 300 BHN

Surface hardness for gear = 200 BHN

Example 4.18

Design a pair of helical gears to transmit a power of 20kW from a shaft running at 1500 rpm to a parallel shaft to be run at 450 rpm. Suggest suitable surface hardness for the gear pair.

(VTU, Jan/Feb 2005, Jan/Feb 2006)

Data :

$$P = N = 20\text{ kW } n_1 = 1500\text{ rpm}; n_2 = 450\text{ rpm};$$

Solution :

Assume

(i) Pressure angle in the normal plane $\alpha_n = 20°$ FD

(ii) Medium shock and 8 to 10 hours duty per day

∴ From Table 2.33 (Old DDHB); **Table 23.13 (New DDHB)** Service factor $C_s = 1.5$

(iii) Helix angle $\beta = 23°$

(iv) Pinon material as 0.40 to 0.50% Carbon steel untreated

∴ From Table 2.57 (Old DDHB); **Table 23.48 (New DDHB)**, $\sigma_{01}, = 69.6$ N/mm²

(v) Scant lubrication but regular inspection

∴ From Table 2.56 (Old DDHB); **Table 23.47 (New DDHB)**, Wear lubrication factor $C_w = 1.25$

Select the minimum number of teeth on pinion to avoid interference for 20° full depth involute system (Referring Table 2.6 (Old DDHB): Table 23.6 (New DDHB)) $z_1 = 18$

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

∴ Number of teeth on gear $z_2 = \frac{n_1}{n_2} \cdot z_1 = \frac{1500}{450} \times 18 = 60$

Formative number of teeth $z_v = \frac{z}{\cos^3 \beta}$ ---- 2.285 (Old DDHB); 23.285 (New DDHB)

∴ Formative number of teeth on pinion $z_{1v} = \frac{z_1}{\cos^3 \beta} = \frac{18}{\cos^3 23} = 23.078$

Formative number of teeth on gear $z_{2v} = \frac{z_2}{\cos^3 \beta} = \frac{60}{\cos^3 23} = 76.926$

Lewis form factor for 20° full depth involute system $y = 0.154 - \frac{0.912}{z_v}$ ---- 2.98 (Old); 23.116 (New)

∴ Form factor for pinion $y_1 = 0.154 - \frac{0.912}{z_{1v}} = 0.154 - \frac{0.912}{23.078} = 0.11448$

Form factor gear $y_2 = 0.154 - \frac{0.912}{z_{2v}} = 0.154 - \frac{0.912}{76.926} = 0.14214$

To select the gear material equate $\sigma_{01}\, y_1$ to $\sigma_{02}\, y_2$

i.e., $69.6 \times 0.11448 = \sigma_{02} \times 0.14214$

∴ $\sigma_{02} = 56.056$ N/mm²

Now from Table 2.57 (Old DDHB); **Table 23.48 (New DDHB)**, select the gear material such that its value of σ_{02} must be nearer to 56.1044 N/mm²

Hence select cast steel, ASTM class B, medium as gear material

$$\therefore \sigma_{02} = 51.70 \text{ N/mm}^2$$

(i) Identify the weaker member

Particulars	σ_o N / mm²	y	$\sigma_o y$	Remarks
Pinion	69.6	0.11448	7,9678	
Gear	51.70	0.14214	7.34864	weaker

Since $\sigma_{02} y_2 < \sigma_{01} y_1$, gear is the weaker member. Therefore design should be based on gear.

Design, dimensions and surface hardness for the gear pair are similar to Example 4.16

Example 4.19

Design a pair of helical gear to transmit 12 kW at 2400 rpm of pinion. The velocity ratio required is 4:1, Helix angle is 23°. The centre distance is to be around 300 mm. Pressure angle in the normal plane is 14½° involute. Pinion material is cast steel ASTM Class B. Gear material is cast iron better grade.

Data :

$$P = N = 12 \text{ kW}; \; a = 300 \text{ mm}; \; n_1 = 2400 \text{ rpm}; \; \alpha_n = 14\tfrac{1}{2}°; \; i = 4; \; \beta = 23°$$

Pinion material – Cast steel ASTM Class B

Gear material – C.I better grade.

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\text{Speed of gear } n_2 = \frac{n_1}{i} = \frac{2400}{4} = 600 \text{ rpm}$$

$$\text{Centre distance } a = \frac{d_1 + d_2}{2} = \frac{d_1 + id_1}{2} = \frac{d_1(1+i)}{2}$$

$$\text{i.e., } 300 = \frac{d_1(1+4)}{2}$$

$\therefore$ Pitch circle diameter of pinion $d_1 = 120$ mm

Pitch circle diameter of gear $d_2 = id_1 = 4 \times 120 = 480$ mm

From Table 2.57(Old DDHB); **Table 23.48 (New DDHB)**, for cast steel ASTM Class B $\sigma_{01} = 51.7$ MPa $= 51.7$ N/mm²

For CI better grade $\sigma_{02} = 31 \text{ MPa} = 31 \text{ N/mm}^2$

To identify the weaker member temporarily assume $z_1 = 20$

$$\therefore z_2 = iz_1 = 4 \times 20 = 80$$

Lewis form factor for 14½° involute $y = 0.124 - \frac{0.684}{z_v}$

Formative number of teeth $z_v = \frac{z}{\cos^3\beta}$ ---- 2.285(Old DDHB); **23.285 (New DDHB)**

$$\therefore\ z_{1v} = \frac{20}{\cos^3 23} = 25.64;\quad z_{2v} = \frac{80}{\cos^3 23} = 102.56$$

Lewis form factor for pinion $y_1 = 0.124 - \frac{0.684}{z_{1v}} = 0.124 - \frac{0.684}{25.64} = 0.0973$.

Lewis form factor for gear $y_2 = 0.124 - \frac{0.684}{z_{2v}} = 0.124 - \frac{0.684}{102.56} = 0.1173$

i) Identify the weaker member

Particulars	σ_0 N/mm²	y	$\sigma_o y$	Remarks
Pinion	51.7	0.0973	5.03	
Gear	31	0.1173	3.63	weaker

Since $\sigma_{02}\, y_2 < \sigma_{01}\, y_1$, gear is weaker. Therefore design should be based on gear..

ii) Design

a) Tangential tooth load $F_t = \frac{9550 \times 1000NC_S}{nr} = \frac{9550 \times 1000PC_S}{nr}$ Where r in mm

Tangential tooth load of the weaker member gear $F_{t2} = \frac{9550 \times 1000NC_s}{n_2 r_2} = \frac{9550 \times 1000PC_s}{n_2 r_2}$

Pitch circle radius of pinion $r_2 = \frac{d_2}{2} = \frac{480}{2} = 240\text{ mm}$

Assume medium shock and 8-10 hours duty per day

∴ From Table 2.33(Old DDHB); Table 23.13 (New DDHB), service factor $C_s = 1.5$

$$\text{i.e., } F_{t2} = \frac{9550 \times 1000 \times 12 \times 1.5}{600 \times 240} = 1193.75\text{ N}$$

b) Tangential tooth load from Lewis equation $F_t = \frac{\sigma_o b y p_t C_v \cos\beta}{C_w} = \frac{\sigma_o b y p_t K_v \cos\beta}{C_w}$

---- 2.286(Old); 23.286 (New DDHB)

$$= \frac{\sigma_o b y p_n K_v}{C_w} = \frac{\sigma_o b y p_n C_v}{C_w} \quad \left(\because\ p_t = \frac{p_n}{\cos\beta}\right)$$

∴ Tangential tooth load of the weaker member $F_{t2} = \frac{\sigma_{o_2} b y_2 p_n C_v}{C_w} = \frac{\sigma_{o_2} b y_2 p_n K_v}{C_w}$

Assume scant lubrication but frequent inspection

∴ From Table 2.56 (Old DDHB); **Table 23.47 (New DDHB)**, wear and lubrication factor $C_w = 1.25$

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_2 n_2}{60000} = \frac{\pi \times 480 \times 600}{60000} = 15.08$ m/sec

Velocity factor $C_v = \frac{6}{6 + v_m} = \frac{6}{6 + 15.08} = 0.284635$ since $v_m < 20$ m/sec

---- 2.289 a(Old); 23.289a (New DDHB)

$$\text{Lewis form factor } y_2 = 0.124 - \frac{0.684}{z_2 v} = 0.124 - \frac{0.684 \times \cos^3 \beta}{z_2}$$

$$= 0.124 - \frac{0.684 \times \cos^3 \beta \times m_n}{d_2 \cos \beta} = 0.124 - \frac{0.684 \times \cos^2 23 \times m_n}{480}$$

$$= 0.124 - 1.2074 \times 10^{-3} m_n$$

For face width, $\frac{p_t}{\tan \beta} < b < \frac{20 m_t}{\tan \beta}$

i.e., $\frac{\pi m_n}{\sin \beta} < b < \frac{20 m_n}{\sin \beta}$ ---- 2.277 and 2.280(Old DDHB); 23.277 and 23.280 (New DDHB)

∴ Select $b = 10\, m_n$

Substituting all these values in Lewis equation

$$1193.75 = \frac{(31)(10 m_n)(0.124 - 1.2074 \times 10^{-3} m_n)(\pi m_n)(0.284635)}{1.25}$$

$$\text{i.e., } 5.383 = 0.124\, m_n^2 - 1.2074 \times 10^{-3} m_n^3$$

$$\text{i.e., } m_n^2 - 9.737 \times 10^{-3} m_n^3 \geq 43.41$$

Trial : 1

Select module $m_n = 6$ mm [select standard module from Table 2.3 (Old DDHB); Table 23.3 (New DDHB)]

$$\therefore 6^2 - 9.737 \times 10^{-3} \times 6^3 \geq 43.41$$

$$33.9 < 43.41$$

∴ Not suitable.

Trial : 2

Select module $m_n = 8$ mm

$$\therefore 8^2 - 9.737 \times 10^{-3} \times 8^3 \geq 43.41$$

$$59.015 > 43.41$$

Hence suitable ∴ module $m_n = 8$ mm

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{02} K_v)_{all} = 31 \times 0.284635 = 8.824$ N/mm^2

$$\text{Induced stress } \sigma_{ind} = (\sigma_{02}K_v)_{ind} = \frac{F_{t2} \times C_w}{by_2 p_n}$$

---- 2.286(Old DDHB); 23.286 (New DDHB)

$$\text{i.e., } \sigma_{ind} = \frac{1193.75 \times 1.25}{(10 \times 8)(0.124 - 1.2074 \times 10^{-3} \times 8)(\pi \times 8)} = 6.49 \text{ N/mm}^2$$

Since $(\sigma_{02} K_v)_{ind} < (\sigma_{02} K_v)_{all}$, the design is safe.

Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more then the effective force between the meshing teeth

$$\text{Effective force } F_{eff} = \frac{F_{t2}.C_s}{K_v} = \frac{F_{t2}}{K_v} \text{ since } C_s \text{ is already considered}$$

$$= \frac{1193.75}{0.284635} = 4193.97 \text{ N}$$

$$\text{Beam strength of weaker member } F_{b2} = \frac{\sigma_{02} by_2 p_n}{C_w}$$

$$= \frac{(31)(10 \times 8)(0.124 - 1.2074 \times 10^{-3} \times 8)(\pi \times 8)}{1.25} = 5701.42 \text{ N}$$

Since $F_{b2} > F_{eff}$, the design is satisfactory and hence the module in normal plane should be equal to 8 mm

iii) Dimensions

Module in normal plane $m_n = 8$ mm

$$\text{Module in diametral plane } m_t = \frac{m_n}{\cos\beta} = \frac{8}{\cos 23} = 8.69 \text{ mm}$$

Face width $b = 10\,m_n = 10 \times 8 = 80$ mm

$$\text{bmin} = \frac{\pi m_n}{\sin\beta} = \frac{\pi \times 8}{\sin 23} = 64.32 \text{ mm}$$

Since $b > 64.32$ mm, face width $b = 80$ mm

Normal pitch $p_n = \pi m_n = \pi \times 8 = 25.133$ mm

Axial pitch $p_t = \pi m_t = \pi \times 8.69 = 27.3$ mm

$$\text{Number of teeth on pinion } z_1 = \frac{d_1 \cos\beta}{m_n} = \frac{120 \times \cos 23}{8} = 13.8$$

$\therefore$ Number of teeth on pinion $z_1 = 14$

Number of teeth on gear $z_2 = iz_1 = 4 \times 14 = 56$

$$\text{Hence actual pitch circle diameter of pinion } d_1 = \frac{m_n z_1}{\cos\beta} = \frac{8 \times 14}{\cos 23} = 121.67 \text{ mm}$$

$$\text{Actual pitch circle diameter of gear } d_2 = \frac{m_n z_2}{\cos\beta} = \frac{8 \times 56}{\cos 23} = 486.67 \text{ mm}$$

$$\text{Actual centre distance } a = \frac{d_1 + d_2}{2} = \frac{121.67 + 486.67}{2} = 304.17 \text{ mm}$$

Actual tangential tooth load $F_{t2} = \dfrac{9550 \times 1000 \times 12 \times 1.5}{600 \times \dfrac{486.67}{2}} = 1177.34$ N

Actual mean pitch line velocity $v_m = \dfrac{\pi \times 486.67 \times 600}{60000} = 15.2892$ m/sec

From Table 2.1 (Old DDHB); Table 23.1 (New DDHB) for pressure angle 14½° involute system

Addendum $h_a = 1 m_n = 1 \times 8 = 8$ mm

Dedendum $h_f = 1.157\ m_n = 1.157 \times 8 = 9.256$ mm

Working depth $h' = 2\ m_n = 2 \times 8 = 16$ mm

Minimum total depth $h = 2.15\ m_n = 2.15 \times 8 = 17.2$ mm

Tooth thickness $s = \dfrac{\pi}{2} m_n = \dfrac{\pi}{2} \times 8 = 12.566$ mm

Minimum clearance $c = 0.157\ m_n = 0.157 \times 8 = 1.256$ mm

Addendum circle diameter of pinion $d_{a1} = d_1 + 2\ h_a = 121.67 + 2 \times 8 = 137.67$ mm---- 2.245(Old); 23.245 (New)

Addendum circle diameter of gear $d_{a2} = d_2 + 2\ h_a = 486.67 + 2 \times 8 = 502.67$ mm---- 2.246(Old); 23.246 (New)

Dedendum circle diameter of pinion $d_{f1} = d_1 - 2\ h_f = 121.67 - 2 \times 9.256 = 103.158$ mm---2.247; 23.247 (New)

Dedendum circle diameter of gear $d_{f2} = d_2 - 2\ h_f = 486.67 - 2 \times 9.256 = 468.158$ mm---- 2.248; 23.248 (New)

For transverse pressure angle $\tan\alpha_t = \dfrac{\tan\alpha_n}{\cos\beta} = \dfrac{\tan 14\frac{1}{2}}{\cos 23}$ ---- 2.221(Old DDHB); 23.221 (New DDHB)

Pressure angle in the diametral or transverse pressure angle $\alpha_t = 15.693°$

Base circle diameter of pinion $d_{b1} = d_1 \cos \alpha_t = 121.67 \times \cos 15.693 = 117.135$ mm---- 2.249; 23.249 (New)

Base circle diameter of gear $db_2 = d_2 \cos \alpha_t = 486.67 \times \cos 15.693 = 468.53$ mm---- 2.250; 23.250 (New DDHB)

Velocity factor $K_v = 0.284635$

Service factor $C_s = 1.5$

Wear and lubrication factor $C_w = 1.25$

iv) Checking

According to Buckingham's equation,

a) Dynamic load $F_d = F_t + \dfrac{21 v_m \left(F_t + bC\cos^2\beta\right)\cos\beta}{21 v_m + \sqrt{F_t + bC\cos^2\beta}}$ ---- 2.297 a(Old DDHB); 23.309a (New DDHB)

From Fig. 2.30 (Old DDHB); Fig. 23.35a (New DDHB)

For $v_m = 15.2892$ m/sec

Error $f = 0.025$

From Table 2.35 (Old DDHB); Table 23.32 (New DDHB)

For pressure angle = 14½° ; steel – CI combination

For error f = 0.025 mm ; C = 192.08 kN/m = 192.08 N/mm i.e. $F_d = 1177.34 + \dfrac{21 \times 15.289\left[1177.34 + 80 \times 192.08 \times \cos^2 23\right]\cos 23}{21 \times 15.289 + \sqrt{1177.34 + 80 \times 192.08 \cos^2 23}} = 10709.05$ N

(b) Wear load

According to Bucking's equation,

$$\text{Wear load } F_w = \frac{d_1 bQK}{\cos^2 \beta}$$ ---- 2.298(Old DDHB); 23.310 (New DDHB)

$$\text{Ratio factor } Q = \frac{2z_2 v}{z_{1v} + z_{2v}} = \frac{2z_2}{z_1 + z_2} = \frac{2 \times 56}{14 + 56} = 1.6$$

For safer design

$$F_w \geq F_d$$

$$\text{i.e., } \frac{d_1 bQk}{\cos^2 \beta} \geq F_d$$

$$\text{i.e., } \frac{(121.67)(80)(1.6)(K)}{\cos^2 23} \geq 9767.9\text{N} \quad \therefore\ K \geq 0.5314 \text{ N/mm}^2$$

From Table 2.40 (Old DDHB); Table 23.37B (New DDHB) for $\alpha = 14½°$ and $K = 0.5314$ N/mm²

Surface hardness for pinion = 300 BHN

Surface hardness for gear = 200 BHN

Example 4.20

A pair of steel helical gears is to transmit 15 kW at 5000 rev/min of the pinion. Both the gears are made of the same material, hardened steel with allowable bending stress of 120 MPa. The gears have to operate at a centre distance of 200 mm. Speed reduction ratio is 4:1. The teeth are 20° full depth involute profile on normal plane. Helix angle is 45°. The gears are manufactured to class III accuracy (precision class). Face width can be taken as 16 times the normal module, If the wear strength has to be more than the dynamic load.

Determine the following : (a) Normal module (b) Transverse module (c) Pressure angle in the transverse plane (d) Number of teeth on pinion and gear (e) Face width (f) Required surface endurence limit

(Note : Lewis bending strength is based on normal module) **VTU July/August 2004**

Data :

$P = N = 15$ kW; $n_1 = 5000$ rpm; $\sigma_{01} = \sigma_{02} = 120$ MPa; $a = 200$ mm;

$i = 4$; $\alpha_n = 20°$ Full depth involute, $\beta = 45°$; Precision class ; $b = 16\, m_n$; $F_w > F_d$

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\text{Speed of gear } n_2 = \frac{n_1}{i} = \frac{5000}{4} = 1250 \text{ rpm}$$

$$\text{Centre distance } a = \frac{d_1 + d_2}{2} = \frac{d_1 + id_1}{2} = \frac{d_1(1+i)}{2}$$

$$\text{i.e., } 200 = \frac{d_1(1+4)}{2}$$

∴ Pitch circle diameter of pinion d_1 = 80mm

Pitch circle diameter of gear $d_2 = id_1 = 4 \times 80 = 320$ mm

As both the gears are of the same material, pinion is the weaker member. Therefore design should be based on pinion.

Lewis form factor for 20° full depth involute $y = 0.154 - \frac{0.912}{z_v}$

Formative number of teeth $z_v = \frac{z}{\cos^3\beta}$ ---- 2.285(Old DDHB); **23.285 (New DDHB)**

Design

(i) Tangential tooth load $F_t = \frac{9550 \times 1000 \times NC_S}{nr} = \frac{9550 \times 1000 \times PC_S}{nr}$ Where r in mm

Tangential tooth load of the weaker member $F_{t1} = \frac{9550 \times 1000NC_S}{n_1 r_1} = \frac{9550 \times 1000PC_S}{n_1 r_1}$

Pitch circle radius of pinion $r_1 = \frac{d_1}{2} = \frac{80}{2} = 40$ mm

Assume medium shock and 8-10 hours duty per day

∴ From Table 2.33 (Old DDHB); **Table 23.13 (New DDHB)**, service factor $C_s = 1.5$

$$\text{i.e., } F_{t1} = \frac{9550 \times 1000 \times 15 \times 15}{5000 \times 40} = 1074.375 \text{ N}$$

(ii) Tangential tooth load from Lewis equation $F_t = \frac{\sigma_o byp_t C_v \cos\beta}{C_w} = \frac{\sigma_o byp_t K_v \cos\beta}{C_w}$

$$\text{i.e., } F_t = \frac{\sigma_o byp_n C_v}{C_w} = \frac{\sigma_o byp_n K_v}{C_w}$$

$\left(\because\ p_t = \frac{p_n}{\cos\beta} \right.$ also it is given bending strength is based on normal module $\left. \right)$

∴ Tangential tooth load of the weaker member $F_{t1} = \frac{\sigma_{01} by_1 p_n C_v}{C_w} = \frac{\sigma_{01} by_1 p_n K_v}{C_w}$

Assume scant lubrication but frequent inspection

∴ From Table 2.56 (Old DDHB); **Table 23.47 (New DDHB)**, wear and lubrication factor $C_w = 1.25$

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_1 n_1}{60{,}000} = \frac{\pi \times 80 \times 5000}{60{,}000} = 20.944$ m/sec

$$\text{Velocity factor } K_v = C_v = \frac{5.6}{5.6+\sqrt{v_m}} = \frac{5.6}{5.6+\sqrt{20.944}} = 0.5503 \text{ since } v_m > 20 \text{ m/sec}$$

---- 2.289 b(Old DDHB); 23.290a (New DDHB)

$$\text{Lewis form factor } y_1 = 0.154 - \frac{0.912}{z_{1v}} = 0.154 - \frac{0.912 \times \cos^3\beta}{z_1}$$

$$= 0.154 - \frac{0.912 \times \cos^3\beta \times m_n}{d_1 \cos\beta} = 0.154 - \frac{0.912 \times \cos^2 45 \times m_n}{80}$$

$$= 0.154 - 5.7 \times 10^{-3} m_n$$

Substituting all these values in Lewis equation

$$1074.375 = \frac{(120)(16m_n)(0.154 - 5.7\times10^{-3} m_n)(\pi m_n)(0.5503)}{1.25}$$

$$\text{i.e., } 0.4046 = 0.154\, m_n^2 - 5.7 \times 10^{-3} m_n^3$$

$$\text{i.e., } m_n^2 - 0.037 m_n^3 \geq 2.6272$$

Trial : 1 Select module $m_n = 1.5$ mm (select standard module from Table 2.3 (Old); Table 23.3 (New DDHB))

$$\therefore\ 1.5^2 - 0.037 \times 1.5^3 \geq 2.6272$$

$$\text{i.e., } 2.125 < 2.6272$$

$\therefore$ Not suitable

Trial : 2 Select module $m_n = 2$ mm

$$\therefore\ 2^2 - 0.037 \times 2^3 \geq 2.6272$$

$$3.704 > 2.6272$$

Hence suitable.

$\therefore$ Module in the normal plane $m_n = 2$ mm

(iii) Check for the stress

$$\text{Allowable stress } \sigma_{all} = (\sigma_{01}K_v)_{all} = 120 \times 0.5503 = 66.036 \text{ N/mm}^2$$

$$\text{Induced stress } \sigma_{ind} = (\sigma_{01}K_v)_{ind} = \frac{F_{t1} \times C_w}{b y_1 p_n}$$

----2.286(Old DDHB); 23.286 (New DDHB)

$$= \frac{1074.375 \times 1.25}{(16\times2)(0.154 - 5.7\times10^{-3}\times2)(\pi\times2)} = 46.84 \text{ N/mm}^2$$

Since $(\sigma_{01}K_v)_{ind} < (\sigma_{01}K_v)_{all}$, the design is safe.

Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth

$$\text{Effective force } F_{eff} = \frac{F_{t1}C_S}{K_v} = \frac{F_{t1}}{K_v} \text{ since } C_s \text{ is already considered}$$

$$= \frac{1074.375}{0.5503} = 1952.344 \text{ N}$$

Beam strength of weaker member $F_{b1} = \frac{\sigma_{01} b y_1 p_n}{C_w} = \frac{(120)(16\times 2)(0.154 - 5.7\times 10^{-3}\times 2)(\pi \times 2)}{1.25} = 2752.46 \text{ N}$

Since $F_{b2} > F_{eff}$, the design is satisfactory and hence the module in normal plane should be equal to 2 mm

(a) Normal module $m_n = 2\text{mm}$

(b) Transverse module $m_t = \frac{m_n}{\cos\beta} = \frac{2}{\cos 45} = 2.8284 \text{ mm}$

Number of teeth on pinion $z_1 = \frac{d_1 \cos\beta}{m_n} = \frac{80\times \cos 45}{2} = 28.28$

As it is given that the Lewis bending strength is based on normal module, standard module is normal module only. So if the gears have to operate exactly at a centre distence of 200 mm, slightly change the helix angle as 45.573° (select β by trial and error method)

$$\text{ie., } z_1 = \frac{80\times \cos 45.573}{2} = 28$$

(d) ∴ Number of teeth on pinion $z_1 = 28$

Number of teeth on gear $z_2 = iz_1 = 4 \times 28 = 112$

Check for the centre distance

$$d_1 = \frac{m_n z_1}{\cos\beta} = \frac{2\times 28}{\cos 45.573} = 80 \text{ mm.}$$

$$d_2 = \frac{m_n z_2}{\cos\beta} = \frac{2\times 112}{\cos 45.573} = 320 \text{ mm}$$

$$\therefore \text{ Centre distence } a = \frac{d_1 + d_2}{2} = \frac{80+320}{2} = 200 \text{ mm}$$

Hence suitable

∴ Correct Helix angle $\beta = 45.573°$

(c) Pressure angle in the transverse plane $\tan \alpha_t = \frac{\tan \alpha_n}{\cos\beta}$ ---- 23.221 (New DDHB)

$$\text{ie., } \tan \alpha_t = \frac{\tan 20}{\cos 45.513}$$

$$\therefore \alpha_t = 27.4725°$$

(e) Face width $b = 16\, m_n = 16 \times 2 = 32$ mm

$$b_{min} = \frac{p_t}{\tan\beta} = \frac{p_n}{\cos\beta . \tan\beta} = \frac{\pi m_n}{\sin\beta} = \frac{\pi \times 2}{\sin 45.573} = 8.8 \text{ mm}$$

Since $b > 8.8$ mm, safe

∴ Face width $b = 32$ mm

(f) Required surface endurance limit

According to Buckingham's equation

Dynamic load $F_d = F_t + \dfrac{21v_m\left(F_t + bC\cos^2\beta\right)\cos\beta}{21v_m + \sqrt{F_t + bC\cos^2\beta}}$ ---- 2.297a(Old DDHB); **23.309a (New DDHB)**

From Fig 2.29 (Old DDHB); Fig. 23.34a (New DDHB) for precision class and $m_n = 2$ mm

Expected error f = 0.0125 mm

From Table 2.35 (Old DDHB); Table 23.32 (New DDHB) for 20° full depth, steel – steel combination and f = 0.0125 mm

Dynamic factor C = 145 kN/m = 145 N/mm

$$\therefore\ F_d = 1074.375 + \frac{21\times 20.944\left(1074.375 + 32\times 145\cos^2 45.573\right)\cos 45.573}{21\times 20.944 + \sqrt{1074.375 + 32\times 145\cos^2 45.573}} = 3145.5\ \text{N}$$

According to Buckingham's Equation, Wear load $F_w = \dfrac{d_1bQK}{\cos^2\beta}$ ---- 2.298(Old); 23.310 (New DDHB)

$$\text{Ratio factor } Q = \frac{2z_2v}{z_{1v} + z_{2v}} = \frac{2z_2}{z_1 + z_2} = \frac{2\times 112}{28 + 112} = 1.6$$

Given wear strength more than dynamic load

i.e., $F_w \geq F_d$

i.e., $\dfrac{d_1bQK}{\cos^2\beta} \geq F_d$

i.e., $\dfrac{(80)(32)(1.6)(K)}{\cos^2 45.573} \geq 3145.5$

$\therefore$ Load stress factor $K \geq 0.3763$

i.e., $\dfrac{\sigma_{fc}^2 \sin\alpha_n}{0.7E_0} \geq 0.3763$

From Table 2.8 (Old DDHB.Vol.I) Or **Table 2.10 (New DDHB. Vol-I)**

For steel $E_1 = E_2 = 206$ GPa $= 206\times 10^3$ N/mm²

$\therefore$ Equivalent Young's modulus $E_0 = \dfrac{2E_1E_2}{E_1 + E_2} = \dfrac{2\times 206\times 10^3\times 206\times 10^3}{206\times 10^3 + 206\times 10^3} = 206\times 10^3$ N/mm²

$$\therefore\ \frac{\left(\sigma_{fc}^2\right)(\sin 20)}{0.7\times 206\times 10^3} \geq 0.3763$$

$\therefore$ Limiting stress for surface fatigue $\sigma_{fc} = \sigma_{-1C} \geq 398.3$ N/mm²

Example 4.21 :

The following data refer to a helical gear drive

(a) Power transmitted 34kW at 2800 rpm of pinion

(b) Speed reduction ratio 4.5

(c) Helix angle 25°

(d) Material for both pinion and gear is medium carbon steel whose allowable bending stress may be taken as 230 MPa, BHN = 275

(e) Pinion diameter is limited to 125mm. Determine module and face width. Check the design for wear strength against dynamic loading.

Determine also the axial thrust on the shaft. **VTU, Feb. 2002**

Data :

$P = N = 34$ kW; $n_1 = 2800$ rpm; $i = 4.5$; $\beta = 25°$; $\sigma_{01} = \sigma_{02} = 230$ MPa; $d_1 \leq 125$ mm ; BHN = 275

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\text{Speed of gear } n_2 = \frac{n_1}{i} = \frac{2800}{4.5} = 622.22 \text{ rpm}$$

Since $d_1 \leq 125$ mm, take $d_1 = 120$ mm

∴ Pitch circle diameter of gear $d_2 = id_1 = 4.5 \times 120 = 540$ mm. As both the gears are of the same material, pinion is the weaker member. Therefore design should be based on pinion.

Design (Considering module in the normal plane (m_n) as the standard module)

(a) Tangential tooth load $$F_t = \frac{9550 \times 1000 NC_S}{nr} = \frac{9550 \times 1000 PC_S}{nr} \text{ Where r in mm}$$

$$\text{Tangential tooth load of the weaker member } F_{t1} = \frac{9550 \times 1000 NC_S}{n_1 r_1} = \frac{9550 \times 1000 PC_S}{n_1 r_1}$$

$$\text{Pitch circle radius of pinion } r_1 = \frac{d_1}{2} = \frac{120}{2} = 60 \text{ mm}$$

Assume medium shock and 8-10 hours duty per day

∴ From Table 2.33 (Old DDHB); Table 23.13 (New DDHB), service factor $C_s = 1.5$

$$\text{i.e., } F_{t1} = \frac{9550 \times 1000 \times 34 \times 1.5}{2800 \times 60} = 2899.107 \text{ N}$$

(b) Tangential tooth load from Lewis equation

$$F_t = \frac{\sigma_0 byp_t C_v}{C_w}\cos\beta = \frac{\sigma_0 byp_n C_v}{C_w} = \frac{\sigma_0 byp_n K_v}{C_w} \left(\because p_t = \frac{p_n}{\cos\beta}\right)$$

$$\therefore \text{ Tangential tooth load of the weaker member } F_{t1} = \frac{\sigma_{01} by_1 p_n C_v}{C_w} = \frac{\sigma_{01} by_1 p_n K_v}{C_w}$$

Assume scant lubrication but frequent inspection

∴ From Table 2.56 (Old DDHB); Table 23.47 (New DDHB), wear and lubrication factor $C_w = 1.25$

Mean pitch line velocity of the weaker member $v_m = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 120 \times 2800}{60000} = 17.6$ m/sec

Velocity factor $K_v = \frac{6}{6 + v_m} = \frac{6}{6 + 17.6}$ since $v_m < 20$ m/sec ---- 2.289 a(Old); 23.289a (New)

$= 0.25424$

Assume pressure angle in the normal plane $\alpha_n = 20°$ full depth

$$\therefore \text{ Lewis form factor } y_1 = 0.154 - \frac{0.912}{z_1 v} = 0.154 - \frac{0.912 \times \cos^3\beta}{z_1}$$

$$= 0.154 - \frac{0.912 \times \cos^3\beta \times m_n}{d_1 \cos\beta} = 0.154 - \frac{0.912 \times \cos^2 25 \times m_n}{120}$$

$$= 0.154 - 6.2426 \times 10^{-3}\, m_n$$

$$\frac{p_t}{\tan\beta} < b < \frac{20 m_t}{\tan\beta} \quad \text{---- 2.277 and 2.280(Old); 23.277 and 23.280 (New DDHB)}$$

$$\text{i.e., } \frac{\pi m_n}{\sin\beta} < b < \frac{20 m_n}{\sin\beta}$$

∴ Select face width $b = 10 m_n$

Substituting all these values in Lewis equation

$$2899.107 = \frac{(230)(10 m_n)(0.154 - 6.2426 \times 10^{-3} m_n)(\pi m_n)(0.25424)}{1.25}$$

$$1.9727 = 0.154\, m_n^2 - 6.2426 \times 10^{-3}\, m_n^3$$

$$\text{i.e., } m_n^2 - 0.0405\, m_n^3 \geq 12.81$$

Trial : 1

Assume $m_n = 4$ mm [select the standard module from Table 2.3 (Old DDHB); Table 23.3 (New DDHB)]

∴ $4^2 - 0.0405 \times 4^3 \geq 12.81$

$13.4 > 12.81$

∴ Suitable

∴ Module in the normal plane $m_n = 4$ mm

Trial : 2

Take $m_n = 3$ mm

$3^2 - 0.0405 \times 3^3 \geq 12.81$

$7.9065 < 12.81$

Hence not suitable

c) Check for the stress

Allowable stress $\sigma_{all} = (\sigma_{01} K_v)_{all} = 230 \times 0.25424 = 58.4752$ N/mm²

Induced stress $\sigma_{ind} = (\sigma_{01} K_v)_{ind} = \frac{F_{t1} C_w}{b y_1 p_n}$ ---- 2.286 (Old), 23.286 (New DDHB)

$$= \frac{2899.107 \times 1.25}{(10 \times 4)(0.154 - 6.2426 \times 10^{-3} \times 4)(\pi \times 4)} = 55.875 \text{ N/mm}^2$$

As $(\sigma_{01} K_v)_{ind} < (\sigma_{01} K_v)_{all}$, the design is safe. Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than effective force between the meshing teeth.

$$\text{Effective force } F_{t_{eff}} = \frac{F_{t_1} C_S}{K_v} = \frac{F_{t_1}}{K_v} \text{ since } C_S \text{ is already considered}$$

$$= \frac{2899.107}{0.25424} = 11403.033 \text{ N}$$

$$\text{Beam strength of weaker member } F_{b1} = \frac{\sigma_{01} b y_1 p_n}{C_w}$$

$$= \frac{(230)(10\times4)(0.154 - 6.2426\times10^{-3}\times4)(\pi\times4)}{1.25} = 11933.8 \text{ N}$$

Since $F_{b_1} > F_{t_{eff}}$, the design is satisfactory and hence the module in normal plane should be equal to 4 mm

∴ Module in normal plane $m_n = 4$ mm

face width $b = 10\, m_n = 10 \times 4 = 40$ mm

Check :

$$b_{min} = \frac{p_t}{\tan\beta} = \frac{\pi m_n}{\sin\beta} = \frac{\pi\times4}{\sin 25} = 29.73$$

Since $b > 29.73$ mm, safe.

$$\text{Number of teeth on pinion } z_1 = \frac{d_1 \cos\beta}{m_n} = \frac{120\times\cos 25}{4} = 27.2$$

∴ Number of teeth on pinion $z_1 = 28$

Number of teeth on gear $z_2 = iz_1 = 4.5 \times 28 = 126$

$$\text{Actual pitch circle diameter of pinion } d_1 = \frac{m_n z_1}{\cos\beta} = \frac{4\times28}{\cos 25} = 123.58 \text{ mm} < 125 \text{ mm} \quad \therefore \text{ Safe design.}$$

Hence

Module in the normal plane $m_n = 4$mm

Face width $b = 40$ mm

Number of teeth on pinion $z_1 = 28$

Number of teeth on gear $z_2 = 126$

Actual pitch circle diameter of pinion $d_1 = 123.58$ mm

$$\text{Actual tangential tooth load } F_{t_1} = \frac{9550\times1000\times34\times1.5}{2800\times\left(\frac{123.58}{2}\right)} = 2815.12 \text{ N}$$

Checking

$$\text{According to Bucking ham's formula Dynamic load } F_d = F_t + \frac{21 v_m \left(F_t + bC\cos^2\beta\right)\cos\beta}{21 v_m + \sqrt{F_t + bC\cos^2\beta}}$$

From Fig. 2.30 (Old DDHB) ; Fig. 23.35a (New DDHB) for v_m = 17.6 m/sec

Error f = 0.02 mm

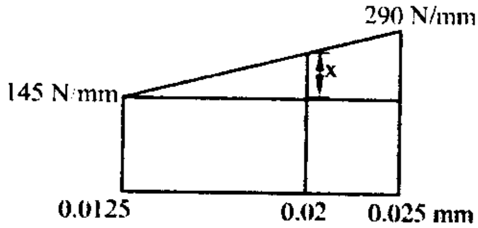
From Table 2.35 (Old DDHB) : **Table 23.32 (New DDHB)**

For 20° full depth, steel – steel combination

For f = 0.0125 mm ; C = 145 kN/m = 145 N/mm

For f = 0.025 mm : C = 290 kN/m = 290 N/mm

By interpolation



$$\frac{x}{0.02-0.0125} = \frac{290-145}{0.025-0.0125}$$

$\therefore$ x = 87 N/mm

$\therefore$ For f = 0.02 mm ;

Dynamic factor C = 145 + 8.7 = 232 N/mm

$$F_d = 2815.12 + \frac{21\times17.6\left[2815.12+40\times232\cos^2 25\right]\cos 25}{21\times17.6+\sqrt{2815.12+40\times232\cos^2 25}} = 10226.26 \text{ N}$$

According to Buckingham's formula

$$\text{Wear load } F_w = \frac{d_1 bQK}{\cos^2\beta}$$ ---- 2.298 (Old DDHB) ; **23.310 (New DDHB)**

$$\text{Ratio factor } Q = \frac{2z_2 v}{z_1 v + z_2 v} = \frac{2z_2}{z_1+z_2} = \frac{2\times126}{28+126} = 1.6364$$

From Table 2.8 [Old DDHB-Volume-I] or Table 2.10 (New DDHB. Vol - I)

For carbon steel Young's modulus $E_1 = E_2 = 206 \times 10^3$ N/mm²

$$\text{Equivalent Young's modulus } E_o = \frac{2E_1E_2}{E_1+E_2} = 206\times10^3 \text{ N/mm}^2$$

Limiting stress for surface fatigue $\sigma_{fac} = (2.75\ H_B - 69)$ MPa ---- 2.291 c (Old DDHB) ; **23.294a (New DDHB)**

$= 2.75 \times 275 - 69 = 687.25$ N/mm²

$$\text{Load stress factor } K = \frac{\sigma_{fc}^2 \sin\alpha_n}{0.7E_o} = \frac{687.25^2 \sin 20}{0.7\times206\times10^3} = 1.12 \text{ N/mm}^2$$

---- 2.299 (Old DDHB) : **23.311 (New DDHB)**

$$\therefore \text{ Wear load } F_w = \frac{(123.58)(40)(1.6364)(1.12)}{\cos^2 25} = 11029.7 \text{ N}$$

As $F_w > F_d$, the design will be satisfactory from the stand point of wear or durability.

Note : If $F_w < F_d$, then

i.e., For safer design $F_w \geq F_d$

$$\text{i.e., } \frac{d_1 bQK}{\cos^2\beta} \geq F_d$$

$\therefore$ Load stress factor $K \geq$ ---- N/mm^2

From Table 23.37 B New (DDHB) for $\alpha = 20°$ and K = ----- N/mm^2 select surface hardness for pinion and surface hardness for gear

Axial thrust on the shaft

Axial thrust force $F_a = F_t \tan\beta = 2815.12 \tan 25 = 1312.7$ N

Design of Double Helical or Herringbone Gear

Example 4.22

A 55 kW motor running at 450 rpm is geared to a pump by means of a double helical gearing. The forged steel pinion on motor shaft has a PCD of around 200 mm and it drives a good grade C.I gear over the pump shaft at 120 rpm. The allowable stress for both pinion and gear material should be taken as 224 N/mm² and 56 N/mm² respectively. Assuming 14½° form teeth with β = 20° and z_1 = 24 design the gears completely. VTU, August 2001

Data :

$P = N = 55$ kW; $n_1 = 450$ rpm; $d_1 \approx 200$ mm; $n_2 = 120$ rpm; $\sigma_{01} = 224$ N/mm^2; $\sigma_{02} = 56$ N/mm^2

$\alpha_n = 14\frac{1}{2}°$; $\beta = 20°$; $z_1 = 24$

Solution :

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1};$$

$$\text{velocity ratio } i = \frac{n_1}{n_2} = \frac{450}{120} = 3.75$$

$\therefore$ Number of teeth on gear $z_2 = iz_1 = 3.75 \times 24 = 90$

$$\text{Formative number of teeth } z_v = \frac{z}{\cos^3\beta}$$ ---- 2.285 (Old DDHB) : 23.285 (New DDHB)

$$\therefore \text{ Formative number of teeth on pinion } z_{1v} = \frac{z_1}{\cos^3\beta}$$

$$= \frac{24}{\cos^3 20} = 28.924$$

$$\text{Formative number of teeth on gear } z_{2v} = \frac{z_2}{\cos^3\beta}$$

$$= \frac{90}{\cos^3 20} = 108.464$$

$$\text{Lewis form factor for } 14\tfrac{1}{2}° \quad y = 0.124 - \frac{0.684}{z_v}$$ ---- 2.97 (Old DDHB) ; 23.115 (New DDHB)

$$\text{Form factor for pinion } y_1 = 0.124 - \frac{0.684}{z_{1v}} = 0.124 - \frac{0.684}{28.924} = 0.1004$$

$$\text{Form factor for gear } y_2 = 0.124 - \frac{0.684}{z_{2v}} = 0.124 - \frac{0.684}{108.464} = 0.1177$$

i) Identify the weaker member

Particulars	σ_o N/mm²	y	σ_o y	Remarks
Pinion	224	0.1004	22.49	
Gear	56	0.1177	6.5912	weaker

Since $\sigma_{o_2} y_2 < \sigma_{o_1} y_1$, gear is weaker. Therefore design should be based on gear.

ii) Design

a) Tangential tooth load $F_t = \dfrac{9550 \times 1000 NC_s}{nr} = \dfrac{9550 \times 1000 PC_s}{nr}$ where r in mm

$\therefore$ Tangential tooth load of the weaker member $F_{t_2} = \dfrac{9550 \times 1000 NC_s}{n_2 r_2} = \dfrac{9550 \times 1000 PC_s}{n_2 r_2}$

From Table 14.4 (Old DDHB Vol-1) or Table 14.7 (New DDHB Vol-I) for pump driven by motor, Load factor i.e., Service factor $C_s = 1.5$

Pitch circle radius of gear $r_2 = \dfrac{d_2}{2} = \dfrac{M_n z_2}{2\cos\beta} = \dfrac{m_n \times 90}{2\cos 20} = 47.888\ m_n$

$$\text{i.e., } F_{t_2} = \frac{9550 \times 1000 \times 55 \times 1.5}{120 \times 47.888 m_n} = \frac{137103.763}{m_n} \quad \text{---(i)}$$

b) Lewis equation for tangential tooth load

$$F_t = \frac{\sigma_o b y p_t C_v \cos\beta}{C_w} = \frac{\sigma_o b y p_n C_v}{C_w} \quad \left(\because p_t = \frac{p_n}{\cos\beta}\right)$$

$$= \frac{\sigma_o b y p_n K_v}{C_w} \text{ ---- 2.286 (Old DDHB) ; \textbf{23.286 (New DDHB)}}$$

Since gear is weaker member

$$F_{t_2} = \frac{\sigma_{02} b y_2 p_n C_v}{C_w} = \frac{\sigma_{02} b y_2 p_n K_v}{C_w}$$

Assume scant lubrication but regular inspection

$\therefore$ From Table 2.56 (Old DDHB) ; **Table 23.47 (New DDHB)** $C_w = 1.25$

For Double helical or Herringbone gear

$$\frac{2.3\pi m_t}{\tan\beta} < b < \frac{30 m_t}{\tan\beta} \quad \text{---- 2.281 and 2.282 (Old DDHB) ; 23.281 and 23.282 (New DDHB)}$$

$$\text{i.e., } \frac{2.3\pi m_n}{\sin\beta} < b < \frac{30 m_n}{\sin\beta}$$

$$\text{i.e., } \frac{2.3\pi m_n}{\sin 20} < b < \frac{30 m_n}{\sin 20}$$

$$21.126\ m_n < b < 87.71\ m_n$$

$\therefore$ Select face width $b = 35\, m_n$

$$\therefore\ F_{t_2} = \frac{(56)(35m_n)(0.1177)(\pi m_n)(K_v)}{1.25}$$

$$= 579.8\, m_n^2\, K_v \qquad \text{---- (ii)}$$

Equating the equations (i) and (ii)

$$579.8\, m_n^2\, K_v = \frac{137103.763}{m_n}$$

$$\therefore\ m_n^3 K_v = 236.47 \qquad \text{---- (iii)}$$

Mean pitch line velocity of the weaker member $v_m = \dfrac{\pi d_2 n_2}{60000}$

$$= \frac{\pi \times m_n z_2 n_2}{60000 \cos\beta}$$

$$= \frac{\pi m_n \times 90 \times 120}{60000 \times \cos 20} = 0.602\, m_n$$

Since approximate pcd of pinion $d_1 = 200\text{mm}$

Approximate module $m_n = \dfrac{200 \cos 20}{24} = 7.83$ mm

Trial : 1 Assume $m_n = 8$ mm

[select standard module from Table 2.3 (Old DDHB) ; Table 23.3 (New DDHB)]

$$\therefore\ v_m = 0.602 \times 8 = 4.816 \text{ m/sec}$$

Velocity factor $C_v = K_v = \dfrac{4.5}{4.5 + v_m} = \dfrac{4.5}{4.5 + 4.816} = 0.483$---- 2.288 (Old) ; **23.288a (New DDHB)**

From equation (iii)

$$(8^3)(0.483) \geq 236.47$$

$$247.316 > 236.47$$

Hence suitable

$\therefore$ Module in normal plane $m_n = 8$ mm

Note :

If it is less, then increase the face width 'b' and check again.

Check

Pitch circle diameter of pinion $d_1 = \dfrac{m_n z_1}{\cos\beta} = \dfrac{8 \times 24}{\cos 20}$

$$= 204.32 \text{ mm}$$

Since the calculated pcd is nearer to 200 mm, the selected module is suitable.

Check for stress, dynamic load, wear load and all other calculations are similar to earlier examples.

Note :

In double helical or herring bone gear except the *face width 'b'* rest everything is similar to helical gear.

Note :

According to Prof. MF Spotts

In order to avoid failure of gear tooth due to bending in the initial stages of design $F_b > F_{eff}$

Introducing factor of safety $F_b = F_{eff}$ (FOS) ---- (i)

Where F_b = Beam strength = $\sigma_b.b.m_n.Y$

F_{eff} = Effective load = $\frac{F_t.C_S}{C_v}$ ---- (ii)

Y = πy, y = Lewis form factor of the weaker member

σ_b = Allowable bending stress = $\frac{\sigma_u}{3}$

σ_u = Ultimate stress

b = Face width

m_n = Module in the normal plane

FOS = Factor of safety

C_S = Service factor

C_v = Velocity factor

$$F_t = \frac{9550 \times 1000N}{nr}$$

r = Weaker member pitch circle radius in mm

n = Speed of the weaker member in rpm

The module (m_n) obtained from equation (i) is based on beam strength

Considering dynamic load according to M.F. Spotts

$$F_{eff} = C_S F_t + F_d \cos \alpha_n \cos \beta \quad \text{---- (iii)}$$

where F_d = Dynamic load = $\frac{en_1z_1br_1r_2}{2530\sqrt{r_1^2 + r_2^2}}$ for steel pinion and steel gear

$F_d = \frac{en_1z_1br_1r_2}{3785\sqrt{r_1^2 + r_2^2}}$ for C.I pinion and C.I gear

$F_d = \frac{en_1z_1br_1r_2}{3260\sqrt{r_1^2 + r_2^2}}$ for steel pinion and CI gear

$e_p + e_g = e$ = Total error (f) ; b = face width

r_2 = Pitch circle radius of gear

z_1 = Number of teeth on pinion

n_1 = Speed of pinion

e_p = Error for pinion ; e_g = error for gear

Considering dynamic load to avoid failure of gear tooth due to bending

$$F_b = F_{eff}\,(FOS)$$

$$= [C_S F_t + F_d \cos\alpha_n \cos\beta]\,FOS \qquad \text{---- (iv)}$$

Wear load $F_w = \dfrac{d_1 bQK}{\cos^2\beta}$

To avoid failure of gear tooth due to pitting

$$F_w > F_{eff}$$

Considering factor of safety

$$F_w = (F_{eff})\,(FOS)$$

Considering dynamic load $F_{eff} = C_S F_t + F_d \cos\alpha_n \cos\beta$

Neglecting dynamic load $F_{eff} = \dfrac{F_t C_S}{C_v}$

$$K = \frac{\sigma_{-1c}^2 \sin\alpha_n}{0.7E_o}$$

$$Q = \frac{2z_{v_2}}{z_{v_1} + z_{v_2}} = \frac{2z_2}{z_1 + z_2}$$

$$E_o = \frac{2E_1E_2}{E_1 + E_2}$$

The gear can also be designed using Prof. M.F. Spott's equations.

REVIEW QUESTIONS

1. Derive an expression for beam strength of a spur gear tooth with standard notations. **VTU, August 2001, January/February 2003**
2. Explain what is meant by Lewis form factor. **BU, August/September 2001**
3. Explain the desirable properties of gear material. **BU, August/September 2001**
4. State the assumptions made in Lewis equation. **BU, January 1994**
5. Explain with sketches internal gear wheel and pinion. **BU, August/September 2001**
6. Explain in brief what is meant by accuracy of gears. **BU, August 1997**
7. What is interference in gears? Explain briefly **BU, March/April 1999**
8. With sketch explain formative or virtual number of teeth applicable to helical gear: also derive an expression for virtual number of teeth interms of helix angle and the actual number of teeth. **VTU, July/August 2004, January 2004**
9. Derive an expression for the load carrying capacity of a helical gear tooth. **BU, December 2003**

EXERCISES

1. Design a pair of spur gears to transmit a power of 20 kW from a shaft running at 1000 rpm to another shaft to be run at 400 rpm. Check the design for dynamic and wear loads.
2. A pair of spur gears is required to transmit 18 kW at 250 rev/min with a speed reduction of 3:1. The centre distance between them is 400 mm. The pinion is made from C 40 steel and gear from cast steel. The teeth are 20° full depth involute. Design the gears.
3. A pair of spur gears transmitting power from a motor to a pump impeller shaft is to be designed with as small distance an possible. The forged steel pinion is to transmit 4 kW at 600 rpm to a cast steel gear with a transmission ratio of 4.5 : 1 and 20° full depth involute teeth are to be used. Design the gears for strength and check for dynamic and wear load.
4. A pair of spur gears is to be used to connect an ore crusher with an electric motor of 100 kW power. The gears are designed for optimum size with the following requirements. Speed of motor = 750 rpm; velocity ratio = 5; Tooth profile = 20° stub; select proper materials. Check the design for dynamic and wear load conditions.
5. Design a pair of equal diameter 20° stub tooth spur gears to transmit 37.3 kW with moderate shock gears to transmit 37.3 kW with moderate shock at 1200 rpm of pinion. The two shafts are parallel and 450 mm apart. Each gear is to be of SAE - 1045 annealed steel.
6. A pair of mating spur gear have 20° full depth involute teeth of 8 mm module. The number of teeth on pinion is 20 and 5 kW will be transmitted at 1500 rpm. The transmission ratio is 5 to 2 calculate :

i) Number of teeth required for gear
ii) Pitch circle diameters
iii) Centre distance
iv) Torque on each shaft and
v) Tangential force.

7. Determine the module and face width of a helical gear tooth for a helical gear pair to transmit a power of 25 kW from a shaft rotating at a speed of 1500 rev/min to a parallel shaft to be run at 300 rev/min maintaining a centre distance of 180 mm.

9. Design a Herringbone drive from a 4 kW steam turbine running at 30,000 rpm to a speed reducer that should run at 2500 rpm. Material static stress may be taken as 105 MPa and endurance stress as 295 MPa. Check the gears for endurance strength and wear.

10. Design a pair of spur gears to transmit a power of 18 kW from a shaft running at 1000 rpm to a parallel shaft to be run at 250 rpm maintaining a distance of 160 mm between the shaft centers. Suggest suitable surface hardness for the gear pair. **VTU, Jan/Feb. 05**

11. Design a pair of helical gears to transmit a power of 20 kW from a shaft running at 1500 rpm to a parallel shaft to be run at 450 rpm. Suggest suitable surface hardness for the gear pair. **VTU, Jan/Feb. 05**

12. A shaft rotating at a speed of 1000 rpm is to transmit a power of 40 kW to a parallel shaft to be rotated at 350 rpm. The distance between the shaft centers is 160 mm. Design a pair of spur gears to connect these two shafts. **VTU, Jan/Feb. 06**

13. Design a pair of helical gears to transmit a power of 30 kW from a shaft rotating at 1500 rpm to a parallel shaft to be rotated at 450 rpm. **VTU, Jan/Feb. 06**

14. a. Give reasons for the selection of involute tooth profile for gears more commonly. While generating less number of teeth on gear wheels the problem of interference prevails in involute tooth gearing, does such a problem really exist in cycloidal tooth gearing?

    b. A pair of spur gears is required to transmit 5 kW of power. The pinion rotates at a speed of 1620 rpm and the gear is required to run 420 rpm. Determine the least number of teeth on the gear and pinion such that velocity ratio does not deviate at all. The tooth form is 20° full depth involute, therefore the number of teeth selected on the pinion should not be less than the theoretical minimum. The permissible stress in the material of the gear and the pinion are 55 MPa and 65 MPa respectively. Design the gears for beam strength only and determine all the proportions of the gearing. **VTU, July 06**

15. A pair of herringbone gears is used to transmit 50 kW power. The pinion rotates at 2800 rpm. The number of teeth on the pinion and gear are 21 and 109, respectively. The tooth form is 20° full depth involute and the helix angle is 25°. The material for the gear is cast steel with a hardness of 150 BHN and for the pinion is steel. The wear and lubrication factor may be taken as 1.15. The normal module employed for the gears is 4 mm and the face width of the gears is 20 times the normal module. Determine the required hardness for the pinion for continuous operation of the drive. Also recommend the class of gears. **VTU, July 06**

16. a) Derive an expression for beam strength of a spur gear tooth with standard notations.

    b) A pair of spur gears has to transmit 20 kW from a shaft rotating at 1000 rpm to a parallel shaft which is to rotate at 310 rpm. Number of teeth on pinion is 31 with 20° full depth involute tooth form. The material for pinion is steel SAE1040 untreated with allowable static stress 206.81 MPa and the material for gear is cast steel 0.20%C untreated with allowable static stress 137.34 MPa. Determine the module and face width of the gear pair. Also find the dynamic tooth load on the gears. Take the service factor as 1.5.

    VTU, Dec. 06/Jan 07

17. A pair of carefully cut helical gears for a turbine has a transmission ratio of 1:10. The teeth are 20° stub involute in the normal plane. Pinion has 25 teeth and rotates at 5000 rpm. Material for pinion and gear is 0.4% carbon steel untreated with allowable static stress of 69.66 MPa. Helix angle = 30°. Power transmitted = 90 kW. Service factor = 1.25. Wear and lubrication factor = 1.25. Determine the module in normal plane and face width of the gears. Suggest suitable surface hardness for the gear pair.

    VTU, Dec. 06/Jan 07

18. a) Derive an expression for beam strength of a spur gear tooth.

    b) Design a pair of spur gears to transmit 15 kW from a shaft rotating at 1000 rpm to a parallel shaft which is to rotate at 310 rpm. Assume number of teeth on pinion 31 and 20° full depth tooth form. The material for pinion is C-40 steel untreated and for gear, cast steel 0.20%C untreated. **VTU, July 07**

19. a) Explain formative number of teeth in helical gears.

    b) A pair of helical gears are to transmit 16 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 1000 rpm and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 Mpa; determine module and face width from static strength considerations and check the gears for wear, given σ_{es} = 618 Mpa. **VTU, July 07**

20. A cast steel pinion rotating at 900 rpm is to drive a cast iron gear at 144 rpm. The static design stresses for pinion and gear materials are 103 MPa and 55 MPa respectively. The teeth are to have standard 20° stub involute profiles and the maximum power to be transmitted is 25 kW. Design the spur gears completely and check for the dynamic and wear loads. The gear surfaces are hardened to BHN 250. Use 16 teeth on the pinion.

    VTU, Dec. 07/Jan. 08

21 a) Explain formative number of teeth in helical gears.

    b) A pair of helical gears with a 23° helix angle is to transmit 2.5 kW at 10,000 rpm of the pinion. The velocity ratio is 4:1. Both gears are to be made of hardened steel with an allowable stress of 100 MPa for each year. The gears are 20° stub and the pinion is to have 24 teeth. Design the gears and determine the required BHN. **VTU, Dec. 07/Jan. 08**

22. A machine running at 360 rpm is driven by 12kW, 1440 rpm motor through a 14½ involute gear. The center distance between the drive is 250mm. The pinion is made up of heat

treated cast steel with allowable stress of 191.2 MPa and 450 BHN. The gear is made of untreated cast steel with allowable static stress of 137.3 MPa and BHN of 300. Assuming gears are working 8 hours/day and subjected to light shocks determine the module, face width and number of teeth on each gear. Also check the design for wear.

VTU, Jun/July 08

23. Design a steel helical gear pair from the following data:

 Power transmitted = 30kW

 Speed of pinion = 1500 rpm

 Velocity ratio = 1:4

 Number of teeth on pinion = 24

 Helix angle $\beta = 30°$

 Static stress for steel $\sigma_{d1} = \sigma_{d2} = 50.7$MPa. Brinell Hardness Number for gear material = 350. Check the design from wear point of view also. **VTU, Jun/July 08**

24. It is required to transmit 25kW of power from shaft running at 1000 rpm to a parallel shaft with speed reduction 2.5 : 1. The centre to centre distance of shaft is to be about 300mm. The material used for pinion is steel (σ_d= 200 N/mm^2, BHN = 250), and for gear is cast iron (σ_d= 180 N/mm^2, BHN = 200). Considering class - II gear with tooth profile 20° full depth involute, design the spur gear and check for dynamic load and wear load.

 VTU, Dec. 08/Jan 09

25. Design a single reduction paralled helical gear speed reducer having speed ratio 5:1. It is to be capable of transmitting the full load rating of 18.75 kW at 1200 rpm of pinion. Use Nickel Chrome steel for pinion and C45 steel for gear. Selected class III precision gears. The helix angle for the gear wheels is 23°. The normal pressure angle of the tooth is 20° and the profile is full depth involute. Check the design for dynamic and wear load.

 VTU, Dec. 08/Jan 09